Middlemen in Search Models with Intensive and Extensive Margins *

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Abstract

This paper studies the emergence of middlemen (intermediation) and its consequences for welfare and redistribution. In Rubinstein and Wolinsky's seminal model on intermediation, middlemen only create value when they have an exogenous advantage in their search speed. I show that this result depends critically on the restriction that middlemen can only carry indivisible quantities of goods. Taking into account the intensive margin of production and allowing them to carry divisible quantities, middlemen with comparatively high bargaining power can create value even when they have a disadvantage in their search cost. While the presence of middlemen can improve welfare, equilibria remain suboptimal due to search and participation externalities. I describe a multi-instrument tax-subsidy scheme that controls participation levels by producers and middlemen and restores efficiency.

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1 Introduction

This paper studies the emergence of intermediation and its consequences for welfare and redistribution in an economy with search frictions by exploring the extensive margin (number of trades) plus the intensive margin (quantity per trade). The importance of intermediation has long been recognized in the empirical and theoretical literature. Empirical work has provided evidence on the contribution of intermediation and variation in its extent across industries. Spulber (1996a) estimates that intermediation comprises over 25% of the U.S. GDP in 1993 by taking into account retail trade (9.33%), wholesale trade (6.51%), finance and insurance (7.28%), and selected services (1.89%). This contribution increased to 34% in 2010, and 44.9% in 2016.[1] Intermediation accounts for a significant share of GDP even in this conservative estimate which assumes intermediation in other sectors is zero. While intermediation is thriving as a whole, there are some markets with a relatively low extent of intermediation and others with a high level. For food sales in the U.S., the share by manufacturers and farmers decreases from 0.46 to 0.39 from 2000 to 2014, while the rest is from supermarkets and other grocery stores.[2] In the market of artwork and antiquities, illicit trade alone amounts to $2.2 billion annually.[3] In the real estate market of the U.S., intermediation accounts for 91% of the total sales. For federal fund markets, the proportion by brokered transaction is approximately 40% throughout 2005-2010.[4]

It has been argued in empirical work that market frictions, including the problems of limited commitment, asymmetric information and difficulties in coordinating trades, are ubiquitous and explains the emergence of middlemen by their superior capacity at addressing some of these obstacles. The measurement of intermediation is a difficult problem. Given endogeneity issues in studying middlemen, empirical work in the area suffers from the lack of a theoretical structure on how market frictions give rise to intermediation. Theoretically, first formalized by Rubinstein and Wolinsky (1987), intermediation is studied in a model with explicit frictions. In their paper, intermediation is an equilibrium and is efficient if and only if intermediaries have an advantage over producers in their ability to search for consumers. A sizable literature studies intermediation using versions of Rubinstein and Wolinsky's model with middlemen having various advantages at mitigating market frictions. Similar to Rubinstein and Wolinsky, a stark restriction in these papers is that goods are indivisible, i.e. quantity per trade is either 0 or 1. This restriction neglects a key feature, namely, that producers and middlemen can mitigate search frictions by adjusting their trading quantities and consequently influence patterns of trade and social welfare. The quantity involved in direct

[1] BEA, U.S. Department of Commerce.
[2] U.S. Department of Agriculture Economic Research Service.
[3] Campbell (2013).
[4] Afonso and Lagos (2014).

trade, i.e. between producers and consumers, and indirect trade, i.e. between middlemen and consumers, is a critical dimension for the study of intermediation, especially when we want to have a model that is empirically and policy relevant.[5]

In this paper, I develop a search-bargaining model which departs from the restriction of indivisible goods and allows quantity per trade to be endogenously decided by agents' strategic choices. In particular, I ask when intermediation emerges in equilibrium, what effects it exerts on welfare and redistribution, and how the extent of intermediation is determined across markets, including the intensive and extensive margins.

New insights are derived on equilibrium and efficiency by taking both margins into account. Equilibrium is inefficient on the two margins and the conventional way of restoring efficiency by setting bargaining powers correctly can only achieve efficiency on one of the two. For the effects of intermediation on welfare and redistribution, I show that welfare can be improved by middlemen even if their only advantage is their bargaining power. This result is different from the existing literature with quantity per trade to be either 0 or 1 (Farboodi, Jarosch and Menzio (2017) and Masters (2007, 2008)) in which intermediation is a rent extraction activity. I find that the effect on welfare depends critically on the quantity restriction per trade. With adjustable quantity, social value created by intermediation not only comes from the number of trades but also the size of each trade. Intuitively, middlemen's bargaining power not only influences the distribution of a total surplus but also crucially affects the trading quantity and therefore the total surplus to be shared. This explains why intermediation in the literature without the intensive margin is welfare-reducing while here welfare losses can be outweighed by gains. Furthermore, I show that welfare is redistributed with intermediation in such a way that, when the search cost is small, welfare is improved for producers and middlemen while consumers are worse off, then as the search cost increases, all agents are better off. These new findings are given by the economics of divisible goods.

The paper explains the existence and welfare effects of intermediation based on two features. One is search costs. Facing the same meeting rate with consumers, if middlemen bear a relatively higher search cost, there can be no intermediation. The other feature is the meeting-specific terms of trade: payment and quantity in a direct trade can be different from those in an indirect trade, and they depend on producers and middlemen's bargaining powers. Given the same total surplus in direct and indirect trades, if middlemen are better are extracting rents from consumers, they can earn a larger share of surplus. On top of that, the intensive margin allows the total surplus

[5] For instance, Philippon (2014), which measures financial intermediation in the U.S. over the past 130 years, finds that the share of financial intermediation in the U.S. GDP varies over time and most of these variations can be explained by corresponding changes in the quantity of intermediated assets.

to be affected by bargaining power through quantity. Therefore, a superior bargaining power not only gives a larger share, but also a larger total surplus.

The environment is as follows. I consider a market with three types of agents, consumers, producers and middlemen, and use a search-bargaining framework related to recent work on models of monetary and asset markets, especially Lagos and Wright (2005). Tractability is maintained by alternating periods of decentralized and centralized exchange: first producers and middlemen choose whether to trade with each other in the wholesale market. It is followed by the retail market which is also frictional. In the retail market, there can be direct trade, indirect trade or both depending on strategic choices of producers and middlemen. They are eligible to participate in retail if they have inventory ready and pay a search cost. After the retail market, agents reconvene in a centralized settlement period. The retail market is a two-sided market where both producers and middlemen serve as sellers. Alternating decentralized and centralized market enables me to study endogenous money holdings by consumers, including how the extent of intermediation is affected by payment frictions and policies. Money and intermediation can be regarded as substitutes since they both ameliorate market frictions, and they also serve as complements in the sense that an increase in inflation lowers money holdings and hurts middlemen.

The first part of the paper characterizes equilibrium. I show that there can be direct trade, indirect trade or both in the retail market. Also I study how participation, production, intermediation and quantity per trade depend on parameters. Equilibrium is such that participation of producers and middlemen are increasing in their own bargaining power with consumers and decreasing in their search costs while quantity per trade is always increasing. This is because of the quantity endogenously depends on their strategic participation decisions. Intuitively, when search costs go up, there are few sellers in the retail market, resulting in a higher buyer-seller ratio and a higher chance for each seller to meet a consumer. Therefore any seller, a producer or a middleman, who chooses to participate, will carry a larger quantity.

Given two dimensions of differences between producers and middlemen, a disadvantage in one dimension, say a higher search cost for middlemen, is not necessarily enough to keep intermediation from emerging. Intuitively, intermediation would emerge if expected net gains (i.e. surplus subtracted by sunk costs) created by bargaining advantage are high enough to cover their disadvantage in search cost. On top of that, an increase in search costs magnifies this effect, in the sense that a higher bargaining power induces a higher quantity per trade. It suggests that, when the search cost for middlemen is larger than for producers, there is still intermediation if middlemen are substantially more skilled at bargaining than producers. Especially when search costs increase, middlemen's advantage in bargaining power would support their participation even with a larger disadvantage in search cost. With quantity per trade being taken

into account, participation of intermediation reacts to parameters differently from the literature with indivisible goods.

The second part of the paper analyzes efficiency of equilibrium and fiscal interventions. This is a natural question since equilibrium in a frictional economy is typically inefficient. I solve a social planner's problem for the optimal numbers of producers and middlemen, and trading quantities. Given random search and bargaining, equilibrium is suboptimal in both margins. Inefficiency in the intensive margin is created by a holdup problem, in the sense that the quantity is too low since costs are sunk before a sale is made. Inefficiency in the extensive margin is associated with the Hosios condition, in the sense that one's participation is not efficient if its contribution to the number of meetings is not correctly reflected in bargaining power. In the literature with indivisible goods/assets, bargaining power is enough to restore efficiency as there is only one margin. This does not work for inefficiency on two margins. Fiscal interventions are designed to restore efficiency by proportional subsidies on quantity per trade, and lump-sum taxes or subsidies on participations.

The third part of the paper explores, without interventions, whether intermediation can be socially beneficial. Although efficiency can be restored with fiscal policy, such interventions need observations of individual's bargaining powers. I want to study, without interventions, if a second best can be achieved in equilibrium with intermediation.

These findings are related to the debate about middlemen contributing to efficiency versus acting as "bloodsuckers" that only buy low and sell high without contributing to value added. I explore effects of intermediation on total welfare and redistribution when middlemen's only advantage is bargaining power. To this end, I compare the total and individual welfare between an economy without middlemen (non-intermediated economy) and an economy with middlemen (intermediated economy). Total welfare depends on the average quantity per trade and the number of sellers. Given the same number of sellers in the two economies, the average quantity per trade is higher in the intermediated economy than with nonintermediated. Now given the same search cost, when it is small, the number of sellers is the same in the two economies, then as it becomes larger, the number of sellers in the intermediated economy is higher because middlemen's participation is encouraged by their superior bargaining power. In fact, total welfare is increasing in the average quantity per trade because of the holdup problem, while changes in the number of sellers can create welfare gains by improving the number of trades but also losses by enlarging total sunk costs. I show that, under some conditions, total welfare is always improved with intermediation and it is redistributed in a way that, when search cost is low, producers and middlemen are better off while consumers are worse off, then as search cost becomes higher, all agents

are better off.[6]

2 Literature Review

In Rubinstein and Wolinsky (1987), goods are indivisible and meeting rates are exogenous. Their main findings are that, firstly intermediation emerges if middlemen are exogenously faster than producers at meeting consumers, and secondly equilibrium is efficient. A sizable literature spurred by Rubintein and Wolinsky's model studies emergence of intermediation in product markets. Yavas (1994) studies intermediaries who trade only one goods with a superior ability at allocating traders of different values. Biglaiser (1993), Li (1998, 1999) and Biglaiser et al. (2017) study intermediaries with superior information on quality of goods. Masters (2007, 2008) analyzes intermediation among agents who are different in their production cost and bargaining power. In Johri and Leach (2002), Shevichenko (2004), and Smith (2004), intermediation caters to end users with heterogenous tastes by holding inventories of a variety of goods. Watanabe (2010, 2013) uses directed search and price posting to study intermediaries with large inventories therefore capable of serving more consumers at a time. Watanabe et al. (2016) study a monopoly middleman as a market maker who provides platforms for traders. The crucial difference between my setup and all these papers is that they restrict the quantity per trade to be either 0 or 1, either with restrictions on sellers' capacity or consumers' demand. As a result, effects on patterns of trade and welfare of intermediation only come from the extensive margin. In my environment, the quantity per trade is endogenous and the effect on this quantity of middlemen's comparatively high bargaining power is at the center of my analysis. Urias (2017) has some commonality in the environment to this paper but focuses on monetary exchange. His paper takes existence of middlemen as given by assuming producers are not allowed to trade directly with consumers, while I endogenize market structure by exploring the determinants and extent of middlemen and additionally analyze their welfare effects.

A related strand of literature studies intermediation in frictional financial markets. Duffie et al. (2005) study intermediation in over-the-counter asset markets. In their paper, investors are heterogenous in their valuation and asset holding position of an indisivible asset. Dealers are good at helping investors get their valuation and asset holding aligned faster. Lagos and Rocheteau (2009) relax the restriction on asset holdings and study how results change with a dispersion of asset positions. Similarly, I allow unrestricted inventory holdings in a product market. In contrast, the motivation

[6] In extensions, I also study the impact of payment and credit frictions as well as directed search with price posting. With directed search and price posting in wholesale and retail markets, equilibrium is optimal on the two margins without interventions. With the impact of imperfect payment and credit, equilibrium results are qualitative robust, while quantitatively different from results in the baseline model with perfect credit. Results are available on my webpage https://gracexungong.weebly.com/

of trade in their paper is driven by heterogeneous valuations in assets. Another paper that explores the importance of divisibility is Golosov et al. (2004). They relax the indivisibility assumption in Wolinsky (1990) and Blouin and Serrano (2001) to study the allocation efficiency and information diffusion with asymmetric information about asset values. Although my goal here is different - to analyze the emergence and welfare effects of intermediation - I share their interest in allowing divisibility and endogenizing the size of trade. Farboodi, Jarosch and Menzio (2017) and Farboodi, Jarosch and Shimmer (2017) develop models with heterogeneous agents in their bargaining power in the former paper and search speed in the latter. My paper like Farboodi, Jarosch and Menzio (2017) explores the welfare effect of intermediation as a rent extraction activity. Where we differ is, I depart from restricting quantity per trade to be 0 or 1 and find that welfare can be improved with intermediation.

Models with similarities to the one proposed here are Nosal et al. (2015, 2016). Nosal et al. (2015) generalize Rubinstein and Wolinsky (1987) by allowing more general bargaining power and search cost. The paper focuses on when intermediation is active and when it is essential. Nosal et al. (2016) allow agents to choose whether to be producers or middlemen. Also they apply this model to asset markets and find existence of multiple equilibria. The main differences with my work are that their papers use indivisible goods and exogenous meeting rate for sellers to meet buyers. The setup in my paper instead is such that goods are fully divisible and endogenously determined, and all meeting rates depend on agents' strategic decisions.[7] These assumptions lead to different implications in terms of the extent of intermediation and its social function, resulting in a higher (lower) participation of middlemen and their welfare-improving effect.

The heterogeneous role of intermediation in terms of extent across industries has been extensively studied in empirical literature. Atack and Passell (1994) take an approach different from Spulber (1996a) to measure the size of intermediation by estimating the employment distribution in the U.S. during 1840-1990 in three sectors - primary (agriculture), secondary (manufacture), and tertiary (services). They show that the proportion of labor force in service sector constantly increased, that is, the ratio of labor in service sector to the total labor in agriculture plus manufactures increased from 0.08 to over 1 in the 150 years.[8] More empirical evidence is well estimated in the literature on intermediation in financial markets. Philippon (2014) measures financial intermediation in the U.S. over the past 130 years and shows variation in its share of GDP over time with a peak just below 9% in 2010. For specific markets, consider the federal fund market for instance, Demiralp et al. (2004) estimate the share of the daily

[7] Some of the literature, e.g. Nosal et al. (2016) or Shevichenko (2004), allows middlemen to hold multiple units, but they still trade $x \in \{0, 1\}$ in each meeting.

[8] Pang and Shi (2010).

volume of fed fund transactions represented by brokered fed funds is about one-third in 2003, Ashcraft and Duffie (2007) report that nonbrokered transactions represented 73% of the volume of fed fund trades in 2005, and Afonso and Lagos (2014) investigate the fed funds intermediated by commercial banks in the last 2.5 hours of the trading session and estimate the proportion by thes brokered transactions to be 40% throughout 2005-2010.

In what follows, Section 3 describes the environment, including the setup, decision rules and terms of trade. Section 4 defines and characterizes the equilibrium which can involve only direct trade, only indirect trade, or both. Section 5 analyzes efficiencies and proposes fiscal interventions. Section 6 explores effects on welfare and redistribution of middlemen when their only advantage is their bargaining power. Section 7 concludes.

3 Environment

3.1 Setup

There is a $[0, 1]$ continuum of agents in three types, C, P and M for consumers, producers and middlemen. The population of each type is fixed and denoted as N_i, $i \in \{C,\ P,\ M\}$, $N_c + N_p + N_m = 1$. Time is discrete and continues forever[9]. In each period there are three sub-periods: a wholesale market, WM, followed by a retail market, RM, both of which are decentralized markets, and finally a frictionless Arrow-Debreu market AD where agents settle debts accumulated in WM or RM. There is a divisible consumption good x traded in WM and RM, which is fully perishable across periods while storable within a period. Let the quantity of good x be denoted as q. This consumption good is only valued by C with utility $u(q)$ and produced by P with cost $c(q)$. Although M cannot produce this consumption good, they can buy it from P and sell it to C. The preference in AD is the same as that in Lagos and Wright (2005), depending on consumption of generic good, X, and labor supply l. All agents use credits as payment instrument when trading in decentralized markets. Then in AD they rebalance their credit and repay their debts, if there is any, and consume good X, which only exists in AD and is produced by their own labor l. While participation of C is passive here, it is interesting to study participation of M and P, and moreover the quantities of trades which can only be considered if decentralized good is *divisible*.

We say that P is active in WM if he is willing to trade with M upon meeting and active in RM if, after failing to meet anyone in WM, he chooses to produce and pay a lump-sum search cost γ_p at the end of WM in order to trade with C in RM. We assume P has a limited amount of raw materials for production so that he can only produce once within a period, therefore if he has traded with M in WM then he is out

[9]For some of the issues addressed here, a three-period model would suffice, but for my extensions on endogenous money and credit, and monetary policy, a model with infinite horizon is critical.

of the market, otherwise he can choose whether to continue to participate in RM to search for C. This assumption is used in the baseline model and relaxed in Appendix. For middlemen, we say that M is active in WM if he is willing to trade with P upon meeting and active in RM if he has traded with P and also pay a lump-sum search cost γ_m such that he has decentralized goods ready to trade with C.

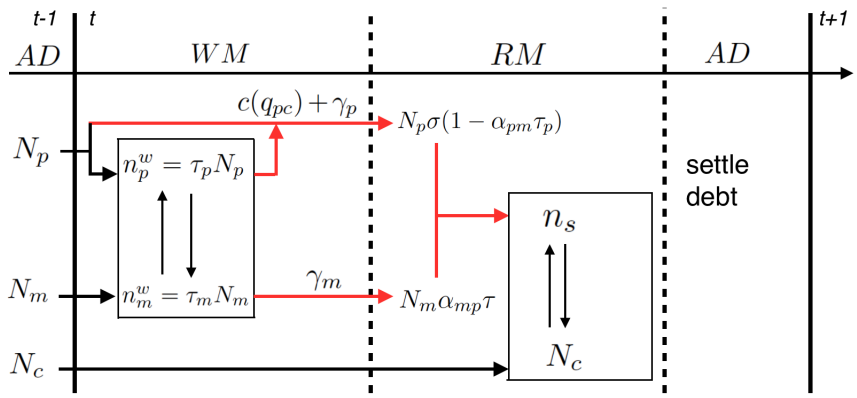

Figure 1:

Participation and bargaining among C, P and M take place in two consecutive two-sided markets WM and RM as shown in Fig.1. In the first sub-period, WM, there are only active P and M. $P(M)$ is active if he/she wants to trade in WM and production happens upon meeting. For inactive P he will produce in this period if choosing to trade with C in the following RM. For inactive M he is out of the market. All production is assumed to take place in WM and it makes P and M who will be active in RM symmetric in the sense that they both bear sunk costs of carrying inventories for RM. When P and M consider whether they should participate in WM, they share the same decision rule, that is, whether the value of their total trading surplus is positive. As P can only produce once within a period, there are two choices to be compared: he can either sell x to M or, instead, produce for C. If he sells x to M, he saves a search cost γ_p and also avoids the search friction in RM. However, he loses the future benefit from potential trade with C in RM which might be more profitable than selling it to M right now. This assumption is relaxed in Appendix by allowing P to produce again after trading with M and it shows that the presented results are qualitatively robust. Now for M, since he can only buy x in WM, a trading decision has to be made based on the tradeoff if buying x from P. If he buys x in WM, he has a chance to sell it to C in RM and share their trading surplus. Nonetheless, the prerequisite is that, he should spend a search cost γ_m to earn such a chance of meeting. Moreover if he fails to meet C in RM, he would bear the cost of search and the cost of inventory purchase since he does not enjoy any utility from consuming x, both of which are sunk at the time of RM. Like in Nosal, Wong and Wright (2015), the search cost is different for M and P, which is an important assumption that influences the role of intermediation. More

interestingly, what makes our model novel and generate new results is that, because of the assumption that good x is divisible, not only the equilibrium measure of participants but also the equilibrium quantities of x that P sells to M and C would be affected by search costs such that there are two channels for search costs γ_p and γ_m to exert their effects on equilibrium sets.

In the second subperiod, RM, there are three types of agents, all of C, active P and active M. By active it means that only those of P and M who have goods and paid search costs before entering RM are eligible to trade with C. Specifically, given search cost γ_p is paid, P is eligible to trade with C if he chooses not to trade with M in WM, or he wants to trade with M but the M he meets does not want to buy from him, or he fails to meet M in WM. For M, he is eligible to trade if he has traded with P in WM and paid search cost before RM starts. As it is assumed that every agent searches only once in each decentralized market, M would not buy from P in RM since has he buys from P in RM there is no chance for him to sell and x is not storable across periods. In RM, C is labeled as buyers (B) while the active P and M are both labeled as sellers (S) such that RM is a two-sided market in which the market tightness is decided by the buyer-seller ratio. After trading activities in WM, the measure of active P and M as well as meeting probabilities need to be updated from WM to RM.

Meeting technology follows a constant return to scale meeting function and depend on the measure of active agents only in each decentralized market. Let n_i^W denote the measure of active type i agents in WM and n_i^R in RM, and α_{ij} the meeting rate at which type i meets type j, then in WM we have

$$\alpha_{pm} = M(1, \frac{n_m^W}{n_p^W}) \tag{1}$$

$$\alpha_{mp} = \frac{\alpha_{pm}}{n_m^W / n_p^W} \tag{2}$$

in RM we have

$$\alpha_{pc} = \alpha_{mc} = \alpha_{sb}(\tfrac{n_c^R}{n_p^R + n_m^R}) = M(1, \frac{n_c^R}{n_p^R + n_m^R}) \tag{3}$$

$$\alpha_{cp} = \alpha_{cm} = \alpha_{bs}(\tfrac{n_c^R}{n_p^R + n_m^R}) = \tfrac{\alpha_{sb}}{n_c^R / (n_p^R + n_m^R)} \tag{4}$$

3.2 Terms of Trade and Decision Rules

We assume that in the baseline model with perfect credit agents split the surplus consistent with generalized Nash or Kalai solutions of many strategic bargaining games.

θ_{ij} is denoted as the bargaining power for type i when meeting type j. In a match the bargaining result is a pair of payment and quantity, denoted as y_{ij} and q_{ji}, meaning that a payment of y_{ij} is transferred from type i (buyer) to type j (seller) in terms of credit for goods sold by j to i. Σ_{ji} is denoted as the total surplus where i is the buyer and j is the seller. Note that since x is fully perishable across periods, M would not purchase from P more than what he would sell to C, implying $q_{mc} = q_{pm}$.

Now we consider decision rules. For C he enters and trades whenever he can so it is trivial. For P and M there are two decisions: one is for both of them on whether to trade in WM , denoted as τ, which will be proved to depend on the same condition for P and M later; the other is for P on whether to participate in RM if he has not traded in WM, denoted as σ. There is no decision for M on whether to participate in RM for the reason that, if he chooses to trade with P and successfully trades with P it is irrational not trading with C, and if he chooses not to trade with P or chooses to but fails to meet P then he has no goods to sell to C and obviously no decision to be made in this case.

Given the decision rules (τ, σ) and the populations of each type (N_c, N_p, N_m), the meeting probabilities can be rewritten as follows,

$$\alpha_{pm} = M(1, \frac{n_m^W}{n_p^W}) = M(1, \frac{N_m\tau}{N_p\tau}) = M(1, \frac{N_m}{N_p}) \tag{5}$$

$$\alpha_{mp} = \frac{\alpha_{pm}}{n_m^W/n_p^W} = \frac{M(1, \frac{N_m}{N_p})}{\frac{N_m}{N_p}} \tag{6}$$

$$\alpha_{pc} = \alpha_{mc} = \alpha_{sb}(\frac{n_c^R}{n_p^R + n_m^R}) = M(1, \frac{N_c}{N_p\sigma(1-\alpha_{pm}\tau) + N_m\alpha_{mp}\tau}) \tag{7}$$

$$\alpha_{cp} = \alpha_{cm} = \alpha_{bs}(\frac{n_c^R}{n_p^R + n_m^R}) = \frac{M(1, \frac{N_c}{N_p\sigma(1-\alpha_{pm}\tau)+N_m\alpha_{mp}\tau})}{\frac{N_c}{N_p\sigma(1-\alpha_{pm}\tau)+N_m\alpha_{mp}\tau}} \tag{8}$$

where in WM the vector of active measure of agents are given by $(n_m^W, n_p^W) = (N_m\tau, N_p\tau)$, and in RM $(n_c^R, n_p^R, n_m^R) = (N_c, N_p\sigma(1-\alpha_{pm}\tau), N_m\alpha_{mp}\tau)$. Notice that $N_p\alpha_{pm} = N_m\alpha_{mp}$. Given divisibility of good x, it is enabled to study not only when middlemen is essential but also what would be the trading quantities.

Let us start from the last subperiod AD. Let $V_i^A(y)$ be the value function for type i agents in AD with credit position of y. V_i^W and $V_i^R(q)$ the value functions in WM and RM, where q is the inventory position of x. Note that y can be either positive, or negative if it is debt.

Firstly we consider the problem for P. In AD the value function for P is

$$
\begin{aligned}
V_p^A(y) &= \max_{X,l}\{U_p(X) - l + \beta V_p^W\} \\
s.t. \quad & X = l - y + T
\end{aligned} \tag{9}
$$

where T is tax, $\beta \in (0,1)$ is the discount rate across periods. We set T to 0 in the baseline model but it is of use in monetary version and in versions with tax/subsidies in RM or WM. Substituting X for l, the FOC is

$$
[X]: \quad U'(X) \quad = 1
$$

and ENV is

$$
V_p^{A'}(y) = 1 \tag{10}
$$

implying that $V_p^S(y)$ is linear in y, and it is also linear in y for the value functions of M and C in SM.

The Bellman equations for P in WM and RM are given by

$$
\begin{aligned}
V_p^W &= \alpha_{pm}[y_{mp} - c(q_{pm}) + V_p^R(0)] + (1-\alpha_{pm})[-c(q_{pc}) + V_p^R(q_{pc})] && (11)\\
V_p^R(q) &= \alpha_{sb}[y_{cp} + V_p^A(0)] + (1-\alpha_{sb})V_p^A(0) - \gamma_p && (12)\\
&= \alpha_{sb}y_{cp} + V_p^A(0) - \gamma_p && (13)
\end{aligned}
$$

The Bellman equations for M in AD, WM and RM are

$$
\begin{aligned}
V_m^A(y) &= \max_{X,l}\{U_m(X) - l + \beta V_m^W && (14)\\
s.t. \quad & X = l - y + T \\
V_m^W &= \alpha_{mp}[V_m^R(q_{pm}) - y_{mp}] + (1-\alpha_{mp})V_m^A(0) && (15)\\
V_m^R(q) &= \alpha_{sb}[V_m^C(q_{-}q_{mc}) + y_{cm}] + (1-\alpha_{sb})V_m^A(0) - \gamma_m \\
&= V_m^A(0) + \alpha_{sb}y_{cm} - \gamma_m && (16)
\end{aligned}
$$

where $q_{pm} = q_{mc}$.

The Bellman equations for C in AD, WM and RM are

$$
\begin{aligned}
V_c^A(y) &= \max_{X,l}\{U_c(X) - l + \beta V_c^W\} && (17)\\
s.t. \quad & X = l - y + T \\
V_c^W &= V_c^R && (18)\\
V_c^R &= \alpha_{bs}[u(q_{pc}) + V_c^A(-y_{cp})] + \alpha_{bs}[u(q_{mc}) + V_c^A(-y_{cm})] \\
& \quad +(1-2\alpha_{bs})V_c^A(0) \\
&= V_c^A(0) + \alpha_{bs}[u(q_{pc}) - y_{cp})] + \alpha_{bs}[u(q_{mc}) - y_{cm}] && (19)
\end{aligned}
$$

Now we analyze the bargaining outcomes in WM and RM. In the baseline model with perfect credit agent i gets a fraction θ_{ij} of total surplus in a trade with j. In WM, there is only bargaining between active M and P given by

$$
\begin{aligned}
\max_{q_{pm}} \quad & V_m^R(q_{pm}) - y_{mp} \\
s.t \quad & V_m^R(q_{pm}) - y_{mp} = \theta_{mp}\Sigma_{mp}
\end{aligned}
$$

where $\Sigma_{pm} = y_{mp} - c(q_{pm}) + V_p^R(0) + V_m^R(q_{pm}) - y_{mp}$. By linearity of $V_i^S(y)$ and $q_{pm} = q_{mc}$, the total surplus of trade between M and P can be rewritten as

$$
\Sigma_{pm} = [\alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - \gamma_m] - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p] \tag{20}
$$

Using the constraint, the objective function is

$$
\max_{q_{pm}} \theta_{mp}\{[\alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - \gamma_m] - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p]\}
$$

FOC for q_{pm} is given by

$$
FOC[q_{pm}]: \ \alpha_{sb}\theta_{mc}u'(q_{pm}) = c'(q_{pm})
$$

Therefore the bargaining solution when M meets P in WM is given by (q_{pm}, y_{mp}) satisfying

$$
\begin{cases}
c'(q_{pm}) & = \alpha_{sb}\theta_{mc}u'(q_{pm}) \\
y_{mp} & = \theta_{mp}[\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) + c(q_{pm}) - \gamma_p] \\
& \quad +\theta_{pm}[\alpha_{sb}\theta_{mc}u(q_{pm}) - \gamma_m]
\end{cases} \tag{21}
$$

In RM, there are two types of meetings, one is for C and P, the other is for C and M. The bargaining problem when C meets P is already solved in the last subperiod WM as all the production takes place in WM. The optimal q_{pc} is given by

$\alpha_{sb}\theta_{pc}u'(q_{pc}) = c'(q_{pc})$. Therefore when C meets P in RM the terms of trade are given by (q_{pc}, y_{cp}) satisfying

$$\begin{cases} c'(q_{pc}) & = \alpha_{sb}\theta_{pc}u'(q_{pc}) \\ y_{cp} & = \theta_{cp}\Sigma_{pc} \end{cases} \tag{22}$$

where $\Sigma_{pc} = u(q_{pc})$. By $q_{mc} = q_{pm}$, the terms of trade between C and M are given by (q_{pm}, y_{cm}) satisfying

$$\begin{cases} c'(q_{pm}) & = \alpha_{sb}\theta_{mc}u'(q_{pm}) \\ y_{cm} & = \theta_{cm}\Sigma_{mc} \end{cases} \tag{23}$$

where $\Sigma_{mc} = u(q_{mc}) = u(q_{pm})$.

Now we consider the decision rules (τ, σ).

Lemma 1. *M wants to trade with P if and only P wants to trade with M.*

The trading decision τ, same for M and P in WM follow the rule given by:

$$\tau = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \quad \textit{if } \Sigma_{pm} \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \tag{24}$$

The decision σ for P depends on the expected payoff for P in RM given by

$$\sigma = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \quad \textit{if } \alpha_{sb}\theta_{pc}\Sigma_{pc} - c(q_{pc}) - \gamma_p \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \tag{25}$$

4 Equilibrium

We are now ready to define an equilibrium. Denote $\mathbf{V}$ as a vector of value functions for C, P *and* M, $\mathbf{y} = (y_{mp}, y_{cp}, y_{cm})$, $\mathbf{q} = (q_{pm}, q_{pc})$, we have

Definition 1. *An equilibrium is a list $< \mathbf{V}, \mathbf{q}, \mathbf{y}, \tau, \sigma >$ such that: $\mathbf{V}$ satisfy value functions given $\mathbf{q}$, $\mathbf{y}$, τ, σ; $(\mathbf{q}, \mathbf{y})$ satisfy the bargaining solutions given $\mathbf{V}$, τ, σ, and (τ, σ) satisfy the best response conditions given $\mathbf{V}$, $\mathbf{q}$, $\mathbf{y}$.*

Given γ_p and γ_m positive, there are four classes of equilibia. The first class of equilibrium is with $\tau = 0$ and $\sigma = 0$, which means both wholesale and retail market shut down, thus neither direct trade from producers to consumers, nor indirect trade from middlemen to consumers. The second class of equilibria is with $\tau = 0$ and $\sigma > 0$, meaning wholesale market shuts down while retail market is open with only producers and no intermediation, and in this case there is only direct trade but no indirect trade. The third class of equilibria is with $\tau > 0$ and $\sigma = 0$, which means both wholesale and retail market are open, and the retail market is operated by intermediation only with no producer, that is, there is only indirect trade but no direct trade. The last class of equilibria is with $\tau > 0$ and $\sigma > 0$, which means both wholesale and retail market are open and the retail market is operated by both producers and intermediation, thus there are both direct and indirect trade.

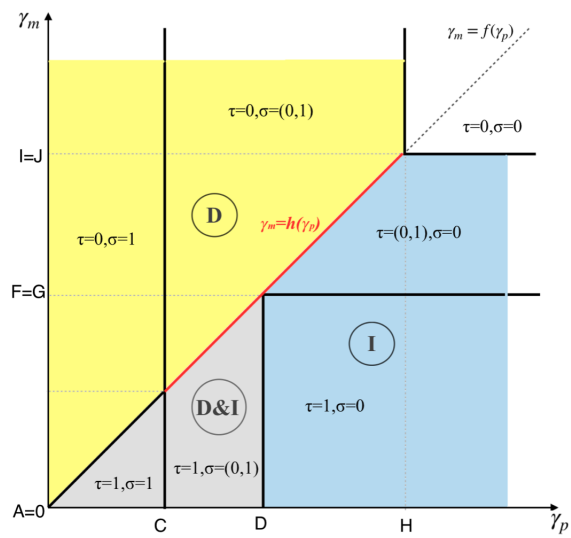

Figure 2: Equilibrium set with $\theta_{mc} = \theta_{pc}$

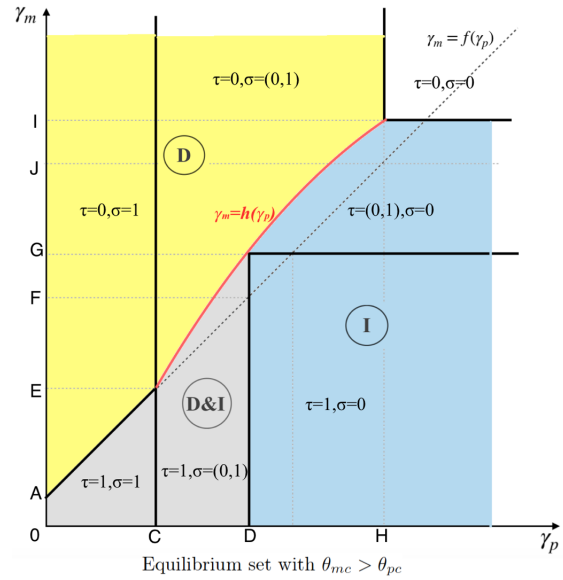

Figure 3: Equilibrium set with $\theta_{mc} > \theta_{pc}$

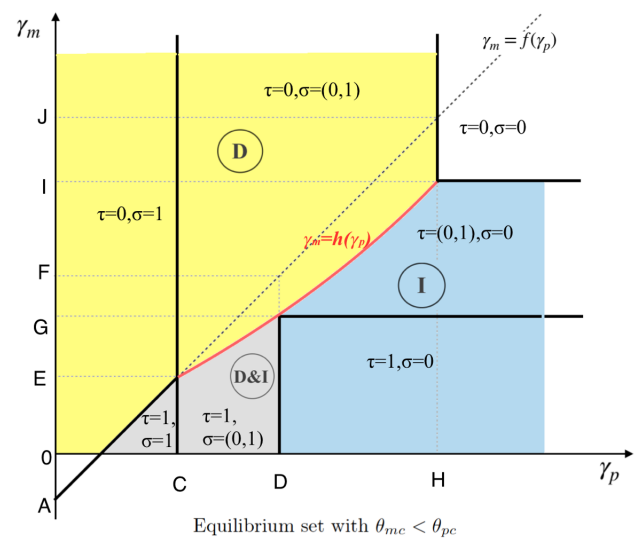

Figure 4: Equilibrium set with $\theta_{mc} < \theta_{pc}$

Before the analysis of equilibrium outcomes, recall that the trading quantity $q_{pc}(\alpha_{sb})$ is solved by $c'(q_{pc}) = \alpha_{sb}\theta_{pc}u'(q_{pc})$ and $q_{mc}(\alpha_{sb})$ by $c'(q_{pm}) = \alpha_{sb}\theta_{mc}u'(q_{pm})$, given α_{sb} being different from case to case depending on (γ_p, γ_m).

To begin the analysis, consider the equilibrium with $\tau = 0$ and $\sigma = 0$. This is an equilibrium when $\Sigma_{pm} \leq 0$ and $\alpha_{pc}\theta_{pc}\Sigma_{pc} - c(q_{pc}) - \gamma_p \leq 0$. Let $\bar{\alpha_{sb}}$ be the value of α_{sb} when $\tau = 0$ and $\sigma = 0$, and $\bar{\alpha_{sb}} = 1$. Then $\tau = 0$ iff $\gamma_m \geq I$ and $\gamma_p \geq H$, where

$$I \equiv \bar{\alpha_{sb}}\theta_{mc}u([q_{mc}(\bar{\alpha_{sb}})] - c[q_{mc}(\bar{\alpha_{sb}})] \tag{26}$$

$$H \equiv \bar{\alpha_{sb}}\theta_{pc}u[q_{pc}(\bar{\alpha_{sb}})] - c]q_{pc}(\bar{\alpha_{sb}})] \tag{27}$$

where $q_{pc}(\bar{\alpha_{sb}})$ and $q_{mc}(\bar{\alpha_{sb}})$ are solved by $c'(q_{pc}) = \bar{\alpha_{sb}}\theta_{pc}u'(q_{pc})$ and $c'(q_{pm}) = \bar{\alpha_{sb}}\theta_{mc}u'(q_{pm})$.

Lemma 2. An equilibrium with $\tau = 0$ and $\sigma = 0$ exists if $\gamma_p \geq H$ and $\gamma_m \geq I$, where H and I are defined in (17) and (18).

Next consider the second class of equilibrium with $\tau = 0$ and $\sigma > 0$, when the wholesale market shuts down thus in the retail market there is no intermediation and only direct trades from producers to consumers. There are two equilibria in this class: one with $\tau = 0$ and $\sigma = 1$, and the other with $\tau = 0$ and $\sigma \in [0, 1]$.

To start with this class consider the pure strategy equilibrium $\tau = 0$ and $\sigma = 1$. This is the case when $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p})$, and $\tau = 0$ iff $\Sigma_{pm} \leq 0$ and $\sigma = 1$ iff $\gamma_p \leq \alpha_{pc}\theta_{pc}\Sigma_{pc} - c(q_{pc})$, so that retail market opens with all producers participating while no intermediation since neither producers nor middlemen want to trade in WM. It is easy to check that $\Sigma_{pm} \leq 0$ iff $\gamma_m \geq f(\gamma_p)$ and $\gamma_p \leq \alpha_{pc}\theta_{pc}\Sigma_{pc} - c(q_{pc})$ iff $\gamma_p \leq C$ where

$$\begin{aligned} f(\gamma_p) &\equiv \gamma_p + \{\alpha_{sb}(\frac{N_c}{N_p})\theta_{mc}u[q_{mc}(\frac{N_c}{N_p})] - c[q_{mc}(\frac{N_c}{N_p})]\} \\ &\quad -\{\alpha_{sb}(\frac{N_c}{N_p})\theta_{pc}u[q_{pc}(\frac{N_c}{N_p})] - c[q_{pc}(\frac{N_c}{N_p})]\} \qquad (28) \\ C &\equiv \alpha_{sb}(\frac{N_c}{N_p})\theta_{pc}u[q_{pc}(\frac{N_c}{N_p})] - c]q_{pc}(\frac{N_c}{N_p})] \qquad (29) \end{aligned}$$

When $\gamma_p = 0$, $\gamma_m = f(\gamma_p) = A$, and when $\gamma_m = 0$, $\gamma_p = f^{-1}(\gamma_m) = B$, where

$$\begin{aligned} A &\equiv \alpha_{sb}(\frac{N_c}{N_p})\theta_{mc}u[q_{mc}(\frac{N_c}{N_p})] - c[q_{mc}(\frac{N_c}{N_p})] \\ &\quad -\{\alpha_{sb}(\frac{N_c}{N_p})\theta_{pc}u[q_{pc}(\frac{N_c}{N_p})] - c[q_{pc}(\frac{N_c}{N_p})]\} \qquad (30) \end{aligned}$$

$$\begin{aligned} B &\equiv \alpha_{sb}(\frac{N_c}{N_p})\theta_{pc}u[q_{pc}(\frac{N_c}{N_p})] - c[q_{pc}(\frac{N_c}{N_p})] \\ &\quad \{\alpha_{sb}(\frac{N_c}{N_p})\theta_{mc}u[q_{mc}(\frac{N_c}{N_p})] - c[q_{mc}(\frac{N_c}{N_p})]\} \qquad (31) \end{aligned}$$

Lemma 3. An equilibrium with $\tau = 0$ and $\sigma = 1$ exists if $\gamma_p \leq C$ and $\gamma_m \geq f(\gamma_p)$, where C and $f(\gamma_p)$ are defined in (19) and (20).

Next consider the equilibrium with $\tau = 0$ and $\sigma \in [0, 1]$ when some but not all of the producers participating in RM. In this case $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p\sigma})$, and $\tau = 0$

iff $\Sigma_{pm} \leq 0$, that is, $\gamma_m \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$, and $\sigma = [0,1]$ iff $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$. By $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$ and $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p\sigma})$, we have $\sigma = \sigma(\gamma_p)$ and thus $\alpha_{sb} = \alpha_{sb}(\gamma_p)$. Substituting $\alpha_{sb}(\gamma_p)$ for α_{sb} in $\gamma_m \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$, then $\tau = 0$ iff $\gamma_m \geq h(\gamma_p)$ where

$$\begin{aligned} h(\gamma_p) &\equiv \gamma_p + \alpha_{sb}(\gamma_p)\theta_{mc}u[q_{pm}(\gamma_p)] - c[q_{pm}(\gamma_p)] - \alpha_{sb}(\gamma_p)\theta_{pc}u[q_{pc}(\gamma_p)] - c[q_{pc}(\gamma_p)] && (32) \\ &= \gamma_p + \alpha_{sb}(\gamma_p)\theta_{mc}u[q_{pm}(\gamma_p)] - c[q_{pm}(\gamma_p)] && (33) \end{aligned}$$

when $\gamma_p = C$, and $\alpha_{sb}(\gamma_p)\theta_{pc}u[q_{pc}(\gamma_p)] - c[q_{pc}(\gamma_p)] = 0$ by $\sigma = [0,1]$. Then $\sigma = 1$, $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p})$, and $\gamma_m \geq h(\gamma_p = C) = E$ where

$$E \equiv \alpha_{sb}(\frac{N_c}{N_p})\theta_{mc}u[q_{pm}(\frac{N_c}{N_p})] - c[q_{pm}(\frac{N_c}{N_p})] \tag{34}$$

and when $\gamma_p = H$, where H is defined in (18), then $\sigma = 0$, $\alpha_{sb} = \bar{\alpha_{sb}}$, and $\gamma_m \geq h(\gamma_p = H) = I$ where

$$I \equiv \bar{\alpha_{sb}}\theta_{mc}u[q_{pm}(\bar{\alpha_{sb}})] - c[q_{pm}(\bar{\alpha_{sb}})] \tag{35}$$

Lemma 4. An equilibrium with $\tau = 0$ and $\sigma \in [0,1]$ exists iff $\gamma_m \geq h(\gamma_p)$ and $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, where $h(\gamma_p)$ is defined in (21) and $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p\sigma})$.

Next consider the third class of equilibria with $\tau > 0$ and $\sigma = 0$. In this case wholesale market is open and in retail market there is only intermediation and no direct trade. There are two candidates to be considered, one with $\tau = 1$ and $\sigma = 0$, and the other with $\tau \in [0,1]$ and $\sigma = 0$. For $\tau = 1$ and $\sigma = 0$ to be an equilibrium, we need $\Sigma_{pm} \geq 0$ and $\gamma_p \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, where $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})$. $\Sigma_{pm} \geq 0$ iff $\gamma_m \leq G$ and $\gamma_p \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$ iff $\gamma_p \geq D$, where

$$G \equiv \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})\theta_{pc}u[q_{pc}(\frac{N_c}{N_m\alpha_{mp}})] - c[q_{pm}(\frac{N_c}{N_m\alpha_{mp}})] \tag{36}$$

$$D \equiv \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})\theta_{pc}u[q_{pc}(\frac{N_c}{N_m\alpha_{mp}})] - c[q_{pc}(\frac{N_c}{N_m\alpha_{mp}})] \tag{37}$$

Lemma 5. An equilibrium with $\tau = 1$ and $\sigma = 0$ exists iff $\gamma_m \leq G$ and $\gamma_p \geq D$, where G and D are defined in (24) and (25).

For $\tau \in [0,1]$ and $\sigma = 0$ to be an equilibrium we need $\Sigma_{pm} = 0$, which holds iff $\gamma_m = \alpha_{sb}\theta_{mc}u[q_{pm}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$, and $\gamma_p \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, where $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}\tau})$. By $\gamma_m = \alpha_{sb}\theta_{mc}u[q_{pm}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$, we have $\tau = \tau(\gamma_m)$, then $\alpha_{sb} = \alpha_{sb}(\gamma_m)$. Substitute $\alpha_{sb}(\gamma_m)$ for α_{sb} in $\gamma_p \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, then $\sigma = 0$ iff $\gamma_p \geq F(\gamma_m)$ where

$$F(\gamma_m) = \gamma_m + \alpha_{sb}(\gamma_m)\theta_{pc}u[q_{pc}(\gamma_m)] - c[q_{pc}(\gamma_m)] \tag{38}$$

When $\gamma_m = G$, then $\tau = 1$, $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})$, and $\gamma_p \geq F(\gamma_m = G) = D$. When $\gamma_m = I$, then $\tau = 0$, $\alpha_{sb} = \bar{\alpha_{sb}}$, and $\gamma_p \geq F(\gamma_m = I) = H$.

Lemma 6. An equilibrium with $\tau \in [0,1]$ and $\sigma = 0$ exists iff $\gamma_m = \alpha_{sb}\theta_{mc}u[q_{pm}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$ and $\gamma_p \geq F(\gamma_m)$, where $F(\gamma_m)$ is defined in (26) and $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}\tau})$.

Next consider the last class of equilibria with $\tau > 0$ and $\sigma > 0$. There are two candidates in this class: one is with $\tau = 1$ and $\sigma = 1$, and the other $\tau = 1$ and $\sigma \in [0,1]$.

First consider the equilibrium with $\tau = 1$ and $\sigma = 1$. This is pure strategy equilibrium in which both WM and RM are open with all of middlemen and producers participating, and $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p})$. $\tau = 1$ iff $\Sigma_{pm} \geq 0$, and $\sigma = 1$ iff $\gamma_p \leq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$. To have these conditions be satisfied, we need $\gamma_p \leq C$ and $\gamma_m \leq f(\gamma_p)$.

Lemma 7. An equilibrium with $\tau = 1$ and $\sigma = 1$ exists iff $\gamma_m \leq f(\gamma_p)$ and $\gamma_p \leq C$, where C is defined in (20) and $f(\gamma_p)$ in (19).

For the other equilibrium in this class $\tau = 1$ and $\sigma \in [0,1]$, $\tau = 1$ iff $\Sigma_{pm} \geq 0$, that is, $\gamma_m \leq \alpha_{sb}\theta_{mc}u[q_{pm}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$ and $\sigma \in [0,1]$ iff $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, where $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p\sigma(1-\alpha_{pm})+n_m\alpha_{mp}})$. This is an equilibrium when WM is open with all middlemen and producers participating while some but not all of those producers who fail to meet a middleman choose to participate in RM. By $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, we have $\sigma = \sigma(\gamma_p)$, then $\alpha_{sb} = \alpha_{sb}(\gamma_p)$. Substituting $\alpha_{sb}(\gamma_p)$ for α_{sb} in $\gamma_m \leq \alpha_{sb}\theta_{mc}u[q_{pm}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$, then $\tau = 1$ iff $\gamma_m \leq g(\gamma_p)$ where

$$g(\gamma_p) \equiv \gamma_p + \alpha_{sb}(\gamma_p)\theta_{mc}u[q_{pm}(\gamma_p)] - c[q_{pm}(\gamma_p)] \tag{39}$$

When $\gamma_p = C$, then $\sigma = 1$, $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p})$, and $\gamma_m \leq g(\gamma_p) = E$. And when $\gamma_p = D$, then $\sigma = 0$, $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})$, and $\gamma_m \leq g(\gamma_p) = G$.

Lemma 8. An equilibrium with $\tau = 1$ and $\sigma \in [0,1]$ exists iff $\gamma_m \leq g(\gamma_p)$ and $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, where $g(\gamma_p)$ is defined in (27) and $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p\sigma(1-\alpha_{pm})+n_m\alpha_{mp}})$.

For completeness we have two other equilibria, $\tau \in [0,1]$ and $\sigma = 1$, and the other $\tau \in [0,1]$ and $\sigma \in [0,1]$, but both of them are possible only for a measuer 0 set of parameters.

Proposition 1. *With γ_p and γ_m both positive, equilibrium exists and is generically unique. The equilibrium set is as shown in Fig. 2.3 and 4 for the cases when $\theta_{pc} = \theta_{mc}$, $\theta_{pc} < \theta_{mc}$ and $\theta_{pc} > \theta_{mc}$. For some parameters intermediation is essential.*

Lemma 9. *$\gamma_m = g(\gamma_p)$, $\gamma_m = F^{-1}(\gamma_p)$ and $\gamma_m = h(\gamma_p)$ are the same curve while $g(\gamma_p)$ is defined with $\gamma_p \in [C, D]$, $F^{-1}(\gamma_p)$ with $\gamma_p \in [D, H]$ and $h(\gamma_p)$ with $\gamma_p \in [C, H]$, and $h'(\gamma_p) > 0$.*

As shown in the graphs, intermediation is essential - i.e. some allocation can not be achieved without intermediation when γ_p is too large even if producers have an advantage over middlemen in terms of bargaining power. What is more important and novel in this model is that, because of divisibility of goods and endogenous α_{ij}, Fig.2, 3 and 4 display how the equilibrium regimes are different because of advantage(or disadvantage) in bargaining power for middlemen over producers when trading with consumers. These results would not be demonstrated without neither divisibility of goods or endogenous meeting technology.

Consider the equilibrium 1 regime when at least one decentralized market is open. Firstly, position of $\gamma_m = f(\gamma_p)$ relative to $\gamma_m = \gamma_p$ is different according to the relative magnitute of θ_{pc} and θ_{mc}. This can also be reflected from the intercept value A of $\gamma_m = f(\gamma_p)$ being different. When $\theta_{pc} = \theta_{mc}$, then $\gamma_m = f(\gamma_p) = \gamma_p$, imply that with the same bargaing power for P and M, trading decision $\tau$is always 0 as long as middlemen has advantage in search cost over producers. When $\theta_{pc} < \theta_{mc}$, then $\gamma_m = f(\gamma_p)$ always is above $\gamma_m = \gamma_p$, and the area in between implies that, with an advantage in bargaining power, middlemen want to trade with producers even if they bear higher search cost than producers. When $\theta_{pc} > \theta_{mc}$, then $\gamma_m = f(\gamma_p)$ is below $\gamma_m = \gamma_p$, and the area in between implies that the advantage in search cost does not necessarily make middlemen be willing to trade since it is eroded by the disadvantage in lower bargaining power compared with producers. Secondly, while we can say that $\tau = 0$ as long as $\gamma_m > f(\gamma_p)$ for equilibria with pure strategy, we cannot conclude the same for equilibria with mixed strategy. The curve that seperating mixed equilibria

with $\tau = 0$ from those with $\tau > 0$ is $\gamma_m = h(\gamma_p)$, and it is also different in Fig.2, 3 and 4. In Fig.2 with $\theta_{pc} = \theta_{mc}$, $\gamma_m = h(\gamma_p)$ overlap with $\gamma_m = f(\gamma_p)$ and $\gamma_m = \gamma_p$, implying as long as middlemen and producers share the same bargaining power in a meeting with a consumers, they always want to trade in WM as long as middlemen has advantage in search cost. In Fig.3 with $\theta_{pc} < \theta_{mc}$, $\gamma_m = h(\gamma_p)$ is above $\gamma_m = f(\gamma_p)$, extending the area of mixed stratege with $\tau > 0$ from what is below $\gamma_m = f(\gamma_p)$. In Fig.4 with $\theta_{pc} < \theta_{mc}$, $\gamma_m = h(\gamma_p)$ is below $\gamma_m = f(\gamma_p)$, extending the area of mixed stratege with $\tau = 0$ from what is above $\gamma_m = f(\gamma_p)$.

In summary, under the symmetric assumptions for middlemen and producers in bearing sunk cost of carring inventory and meeting consumers at the same speed in RM, it is shown in Fig.2, 3 and 4 that bargaining powers and search costs can still give rise to essentiality of intermediation. More importantly, bargaining power and search cost can now exert effect via two channels: intensive margin and extensive margin. Therefore there are two curves, $\gamma_m = f(\gamma_p)$ and $\gamma_m = h(\gamma_p)$, served as thresholds on the existence of intermediation, and these two curves are differenct unless the bargainng power is the same for producers and middlemen. All these differences are contributed by economics of divisibility of goods and endogenous meeting techonology.

Lemma 10. *When $\theta_{mc} = \theta_{pc}$, then $A = 0$, $F = G < I = J$. When $\theta_{mc} > \theta_{pc}$, then $A > 0$, $F < G < J < I$. When $\theta_{mc} < \theta_{pc}$, then $A < 0$, $G < F < I < J$. Also given $\gamma_p \in [C, H]$, $h(\gamma_p)$ is part of an upward-sloping curve, $h'(\gamma_p) > 1$ if $\theta_{mc} > \theta_{pc}$, $h'(\gamma_p) < 1$ if $\theta_{mc} < \theta_{pc}$, and $h'(\gamma_p) = 1$ if $\theta_{mc} = \theta_{pc}$.*

Although the results given by the divisibility of goods is involved in the above graphical analysis, the attention is focused on the outcome of equilibrium participation decision, which is the entensive margin. What would be interesting to consider in detail with divisible goods is the intensive margin, that is, quantity of goods (q_{pm}, q_{pc}) traded in WM and RM. Also notice that compared with the equilibrium outcome with indivisible goods, the number of candidate equilibria with mixed strategies are extended with divisible goods since now the equilibrating force is allowed to work through two channels: both the number of active agents as well as the trading quantities are adjustable.

To start with the analysis on trading quantities (q_{pm}, q_{pc}), consider (q_{pm}, q_{pc}) in pure strategy equilibria $(\tau, \sigma) \in \{(1,1), (0,1), (1,0), (0,0)\}$. For $(\tau, \sigma) = (1,1)$ and $(\tau, \sigma) = (0,1)$, the meeting probabilities are the same, $\alpha_{sb} = \alpha_{sb1} \equiv M(1, \frac{N_c}{N_p})$, and $(q_{pm}, q_{pc}) = (q_{pm1}, q_{pc1})$ where q_{pm1} is solved by $\alpha_{sb1}\theta_{mc}u'(q_{pm1}) = c'(q_{pm1})$, q_{pc1} by $\alpha_{sb1}\theta_{mc}u'(q_{pm1}) = c'(q_{pm1})$. For $(\tau, \sigma) = (1,0)$, $\alpha_{sb} = \alpha_{sb2} \equiv M(1, \frac{N_c}{N_m \alpha_{mp}})$, where $\alpha_{mp} = M(\frac{N_p}{N_m}, 1)$, and, similarly, $(q_{pm}, q_{pc}) = (q_{pm2}, q_{pc2})$ where q_{pm2} is solved by $\alpha_{sb2}\theta_{mc}u'(q_{pm2}) = c'(q_{pm2})$, q_{pc2} by $\alpha_{sb2}\theta_{mc}u'(q_{pm2}) = c'(q_{pm2})$. For $(\tau, \sigma) = (0,0)$, $\alpha_{sb} = \bar{\alpha_{sb}}$, and $(q_{pm}, q_{pc}) = (\bar{q_{pm}}, \bar{q_{pc}})$

where$\bar{q_{pm}}$ is solved by $\bar{\alpha_{sb}}\theta_{mc}u'(\bar{q_{pm}}) = c'(\bar{q_{pm}})$, $\bar{q_{pc}}$ by $\bar{\alpha_{sb}}\theta_{mc}u'(\bar{q_{pc}}) = c'(\bar{q_{pc}})$.

In pure strategy equilibria, obviously $\alpha_{sb1} \leq \alpha_{sb2} \leq \bar{\alpha_{sb}}$, and it is proved in the Appendix that:

Proposition 2. $\frac{\partial q_{pm}}{\partial \theta_{mc}} \geq 0$, $\frac{\partial q_{pc}}{\partial \theta_{pc}} \geq 0$, $\frac{\partial q_{ij}}{\partial \alpha_{sb}} \geq 0$, $\frac{\partial q_{ij}}{\partial (\alpha_{sb}\theta_{ij})} \geq 0$, $ij \in \{pc, pm\}$

Therefore $q_{pm1} \leq q_{pm2} \leq \bar{q_{pm}}$, and $q_{pc1} \leq q_{pc2} \leq \bar{q_{pc}}$.

Next consider (q_{pm}, q_{pc}) in mixed strategy equilibria. For $(\tau, \sigma) = (1, [0, 1])$, $\alpha_{sb} = \alpha_{sb3} \equiv M(1, \frac{N_c}{N_p\sigma(1-\alpha_{pm})+N_m\alpha_{mp}})$. Let q_{pm3} and q_{pc3} denote the quantities in this equilibrium. It is easy to check that when $\gamma_p = C$, then $\sigma = 1$, $\alpha_{sb3} = \alpha_{sb1}$, $q_{pm3} = q_{pm1}$ and $q_{pc3} = q_{pc1}$. Generally q_{pm3}, q_{pc3} and σcan be solved in terms of γ_p by

$$\begin{cases} \gamma_p = \alpha_{sb3}\theta_{pc}u(q_{pc3}) - c(q_{pc3}) \\ \alpha_{sb3}\theta_{pc}u'(q_{pc3}) = c'(q_{pc3}) \\ \alpha_{sb3}\theta_{mc}u'(q_{pm3}) = c'(q_{pm3}) \end{cases} \tag{40}$$

For $(\tau, \sigma) = (0, [0, 1])$, $\alpha_{sb} = \alpha_{sb4} \equiv M(1, \frac{N_c}{N_p\sigma})$. Let q_{pm4} and q_{pc4} denote the quantities in this case. When $\gamma_p = C$, then $\sigma = 1$, $\alpha_{sb} = \alpha_{sb1}$, $q_{pm4} = q_{pm1}$, $q_{pc4} = q_{pc1}$. When $\gamma_p = H$, $\sigma = 0$, $\alpha_{sb} = \bar{\alpha_{sb}}$, $q_{pm4} = \bar{q_{pm}}$, and $q_{pc4} = \bar{q_{pc}}$. Generally q_{pm4}, q_{pc4} and σcan be solved in terms of γ_p by

$$\begin{cases} \gamma_p = \alpha_{sb4}\theta_{pc}u(q_{pc4}) - c(q_{pc4}) \\ \alpha_{sb4}\theta_{pc}u'(q_{pc4}) = c'(q_{pc4}) \\ \alpha_{sb4}\theta_{mc}u'(q_{pm4}) = c'(q_{pm4}) \end{cases} \tag{41}$$

For $(\tau, \sigma) = ([0, 1], 0)$, $\alpha_{sb} = \alpha_{sb5} \equiv M(1, \frac{N_c}{N_m\alpha_{mp}\tau})$. Let q_{pm5} and q_{pc5} denote the quantities in this equilibrium. When $\gamma_m = G$, then $\tau = 1$, $\alpha_{sb} = \alpha_{sb2}$, $q_{pm5} = q_{pm2}$, $q_{pc5} = q_{pc2}$. When $\gamma_m = I$, $\tau = 0$, $\alpha_{sb} = \bar{\alpha_{sb}}$, $q_{pm5} = \bar{q_{pm}}$, and $q_{pc5} = \bar{q_{pc}}$. Generally q_{pm5}, q_{pc5} and τ can be solved in terms of γ_m by

$$\begin{cases} \gamma_m = \alpha_{sb5}\theta_{mc}u(q_{pm5}) - c(q_{pm5}) \\ \alpha_{sb5}\theta_{pc}u'(q_{pc5}) = c'(q_{pc5}) \\ \alpha_{sb5}\theta_{mc}u'(q_{pm5}) = c'(q_{pm5}) \end{cases} \tag{42}$$

In the class of mixed stategy equilibria, when $\sigma \in [0, 1]$, it is proved that:

Proposition 3. *When* $\sigma \in [0, 1]$, $\frac{\partial \sigma}{\partial \gamma_p} < 0$, $\frac{\partial \alpha_{sb}}{\partial \gamma_p} > 0$, $\frac{\partial q_{pm}}{\partial \gamma_p} > 0$, *and* $\frac{\partial q_{pc}}{\partial \gamma_p} > 0$.

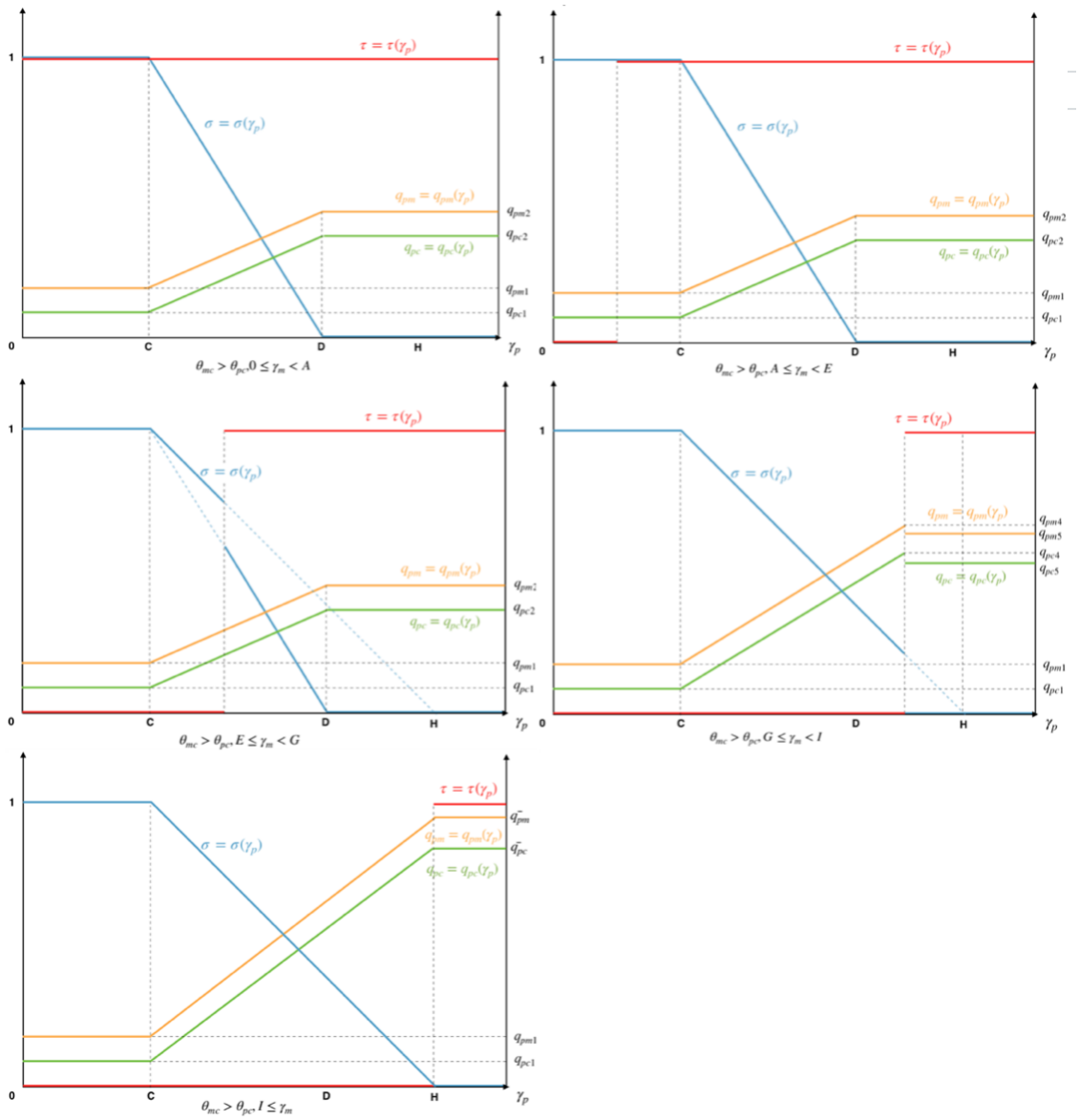

Figure 5: Changes in σ, τ and trade volumes *w.r.t.* γ_p when $\theta_{mc} > \theta_{pc}$, given γ_m

Proposition 4. *When* $\tau \in [0,1], \frac{\partial \tau}{\partial \gamma_m} < 0,\ \frac{\partial \alpha_{sb}}{\partial \gamma_m} > 0,\ \frac{\partial q_{pm}}{\partial \gamma_m} > 0,$ *and* $\frac{\partial q_{pc}}{\partial \gamma_m} > 0.$

Based on Fig.3 with $\theta_{mc} > \theta_{pc}$ as an example, Fig.5 illustrates how q_{pm}, q_{pc}, $\tau$and σ change in response to γ_p for a given γ_m in equilibrium. Based on the range from which γ_m is choosen, there are five possible outcomes describing the relationships. Generally, intensive margin and extensive margin move in the opposite directions when in mixed strategy equilibria. Also in some cases there are discrete changes in intensive and/or extensive margin.

Speciafically the left one at the top illustrates the outcome when $0 \le \gamma_m < A$. The right one at the top illustrates the outcome when $A \le \gamma_m < E$, which is the same as

when $0 \leq \gamma_m < A$ except $\tau = 0$ for small γ_p , and at the point of $\gamma_p = f^{-1}(\gamma_m)$ there is a discrete change of τ, jumping upward from 0 to 1.

The left one in the middle is for the outcome when $E \leq \gamma_m < G$. In this case, there are discontinuities in both extensive margin at the same point of γ_p, while not in intensive margin. Given γ_m, the value of equilibrium σ drops down from σ_4 to σ_3 at the point when $\gamma_p = h^{-1}(\gamma_m)$, where σ_4 is the value of σ in equilibrium with $\tau = 0$ and $\sigma \in [0, 1]$, and σ_3 is the value of σin equilibrium with $\tau = 1$ and $\sigma \in [0, 1]$. When $\gamma_m = E$, $\sigma_4 = \sigma_3 = 1$, and when $\gamma_m = G$, $\sigma_4 = \alpha_{pm}$ and $\sigma_3 = 0$. As γ_m increases from E to G, the drop in σ increases from 0 to α_{pm}. Also given γ_m, the value of equilibrium τ jumps up from 0 to 1 at the same point of γ_p when σ drops down.

The right one in the middle shows the outcome when $G \leq \gamma_m < I$. In this case, there are discontinuities in both intensive margin and extensive margin. σ and τ jump at $\gamma_p = h^{-1}(\gamma_m)$ for a given $\gamma_m \in [G, I)$. The value of equilibrium τ jumps up from 0 to 1 at $\gamma_p = h^{-1}(\gamma_m)$ and σ drops to 0. And the discrete drop in σ gets smaller as γ_p increases. Specifically the value of σ before dropping down to 0 is solved by

$$\begin{cases} h^{-1}(\gamma_m) = \alpha_{sb4}\theta_{pc}u(q_{pc4}) - c(q_{pc4}) \\ \alpha_{sb4}\theta_{pc}u'(q_{pc4}) = c'(q_{pc4}) \\ \alpha_{sb4} = \frac{N_c}{N_c + N_p\sigma} \end{cases}$$

In this case, there are also discrete changes in the equilibrium quantity (q_{pc}, q_{pm}). However it is ambigous whether the quantity increases or decreases at the point of discontinuity and this would depend on the specific form of utility and cost function.

The last graph at the bottom in Fig.5 shows the change in intensive and extensive margin with respect to γ_p when $I \leq \gamma_m$.

In summary, the equilibrium σ is 1 when γ_p is small and goes down to 0 as γ_p increases for any given γ_m. For equilibrium τ, middlemen's advantage over producers in search cost is reduced as γ_m increases and thus the set of γ_p supporting $\tau = 1$ shrinks. This result is obvious as shown in Fig.5. Also notice that, as γ_p changes, whenever there is jump in σ there is also a jump in τ in the opposite direction. Moreover, the equilibrium q_{pc} stays the same whenever $\sigma$is constant while q_{pc} moves in the opposite direction with σ when σ changes. As for q_{pm}, although it seems not responsive to changes in τ, it changes whenever q_{pc} changes since both are affected by the market tightness in RM. With divisible good, the search cost not only give rise to endogenously adjustment in the composition of active sellers, but also the adjustment of equilibrium quantities as well such that we can study the change in extensive margin and intensive margin in response to different levels of search costs.

5 Efficiency and Fiscal Intervention

It is natural to study the efficient outcome since the above equilibrium analysis is based on a market with frictions. For a social planner, the problem is given by

$$\max_{\tau^o,\sigma^o,q^o_{pm},q^o_{pc}} \quad N_p\tau^o\alpha^o_{pm}[-c(q^o_{pm})] + N_m\tau^o\alpha_{mp}[\alpha^o_{sb}u(q^o_{pm}) - \gamma_m] \\ +N_p\sigma^o(1-\tau^o\alpha_{pm})[\alpha^o_{sb}u(q^o_{pc}) - c(q^o_{pc}) - \gamma_p] \tag{43}$$

where $\alpha^o_{sb} = M(1, \frac{N_c}{N_p\sigma^o(1-\alpha_{pm}\tau^o)+N_m\alpha_{mp}\tau^o})$. By using $N_p\alpha_{pm}$ to substitute for $N_m\alpha_{mp}$ in the optimization problem and dividing the function by N_p, then it is the same as solving

$$\max_{\tau^o,\sigma^o,q^o_{pm},q^o_{pc}} \quad Z \equiv \tau^o\alpha_{pm}[\alpha^o_{sb}u(q^o_{pm}) - c(q^o_{pm}) - \gamma_m] + \sigma^o(1-\tau^o\alpha_{pm})[\alpha^o_{sb}u(q^o_{pc}) - c(q^o_{pc}) - \gamma_p] \tag{44}$$

Given α^o_{sb}, $q^o_{pm} = q^o_{pc} = q^o$ is solved by

$$\alpha^o_{sb}u'(q^o) = c'(q^o) \tag{45}$$

Qualitatively the graph of efficient set is similar to the graph of equilibrium set when $\theta_{pc} = \theta_{mc}$.

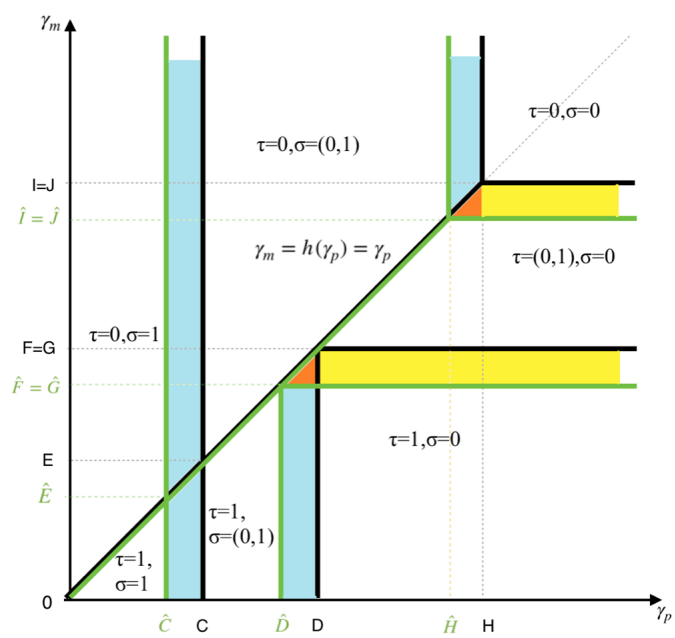

Figure 6: Inefficiency of the extensive margin when $\theta_{mc} = \theta_{pc}$

Proposition 5. *The efficient outcome exists and is generically unique, as shown in Fig.6 with green boundaries.*

To compare equilibrium with efficient outcome, there are two margins to be considered, extensive margin and intensive margin. Alternatively, we can also compare the range of γ_p and γ_m that support the same extensive margin in equilibrium and social planner's problem, and check the difference in intensive margin.

To get some intuition we can set intensive margin in equilibrium to be efficient by controling some parameters and focus on the extensive margin. Recall that q_{pm} is solved by $\alpha_{sb}\theta_{mc}u'(q_{pm}) = c'(q_{pm})$, q_{pc} by $\alpha_{sb}\theta_{pc}u'(q_{pc}) = c'(q_{pc})$, and q^o by $\alpha_{sb}u'(q^o) = c'(q^o)$. By setting $\theta_{pc} = \theta_{mc} = 1$ in equilibrium then $q_{pm} = q_{pc} \equiv q^e = q^o$. The comparison is as shown in Fig.6.

To begin with the analysis of extensive margin, consider the conditions on γ_p and γ_m such that $(\tau, \sigma) = (1, 1)$, and $\alpha_{sb1} = \alpha^0_{sb1}$. In equilibrium, γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \leq \alpha^o_{sb1}u(q^o_1) - c(q^o_1) \\ \gamma_m & \leq \gamma_p \end{cases}$$

In social planner's problem, γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \leq (\alpha^o_{sb1})^2u(q^o_1) - c(q^o_1) \\ \gamma_m & \leq \gamma_p \end{cases}$$

Obviously, to support $(\tau, \sigma) = (1, 1)$, the ranges of γ_p and γ_m in equilibrium are no less than those in efficiency. Therefore, in this simple case when producers and middlemen have full bargaining power, although the intensive margin (q_{pm}, q_{pc}) and extensive margin τ equilibrium are efficient, the extensive margin σ is not. This is an example of too many producers participating in equilibrium since the efficient outcome is $\sigma \in [0, 1]$ while in equilibrium $\sigma = 1$ for $\gamma_p \in [(\alpha^o_{sb1})^2u(q^o) - c(q^o), \alpha^o_{sb}u(q^o) - c(q^o)]$.

Next consider a case of inefficiency when there is too many middlemen participating in equilibrium. Consider the outcome of $(\tau, \sigma) = ([0, 1], 0)$, by which $\alpha_{sb5} = \alpha^o_{sb5}$, then in equilibrium, γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \geq \gamma_m \\ \gamma_m & \in [G, I] \end{cases}$$

In social planner's problem, γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \geq \gamma_m \\ \gamma_m & \in [G^o, I^o] \end{cases}$$

where $G^e = \alpha_{sb2} u(q_{pm2}) - c(q_{pm2})$, $I^e = \bar{\alpha_{sb}} u(\bar{q_{pm}}) - c(\bar{q_{pm}})$, $G^o = (\alpha^o_{sb2})^2 \theta_{mc} u(q^o_{pm2}) - c(q^o_{pm2})$, $I^o = (\bar{\alpha^o_{sb}})^2 \theta_{mc} u(\bar{q^o_{pm}}) - c(\bar{q^o_{pm}})$, and q_{pm}, q_{pc} and σ in equilibrium are the same as the efficient outcomes since $\theta_{pc} = \theta_{mc} = 1$. Nonetheless it is not always the case for τ. To prove this, it is easy to check that $G^o < G^e$, $I^o < I^e$. When $\gamma_m \in \{[G^o, G], [I^o, I]\}$, there would be too many middlemen participating: when $\gamma_m \in [G^o, G]$, $\tau^e = 1$ in equilibrium while $\tau^o \in [0, 1]$ in optimality; when $\gamma_m \in [I^o, G]$, $\tau^e \in [0, 1]$ in equilibrium while $\tau^o = 0$ in optimality.

Next consider a case when there are too many middlemen as well as too many producers participating in equilibrium. Consider $(\tau, \sigma) = (1, 0)$ in equilibrium, then γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \geq D^e \\ \gamma_m & \leq G^e \end{cases}$$

In social planner's problem, γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \geq D^o \\ \gamma_m & \leq G^o \end{cases}$$

where $D^e = \alpha_{sb2} u(q_{pc2}) - c(q_{pc2})$, $D^o = (\alpha^o_{sb2})^2 u(q^o_{pc2}) - c(q^o_{pc2})$. Still intensive margin are efficient given full bargaining power for producers and middlemen in retail market but the extensive margin in not efficient in equilibrium. It is easy to check that $D^o < D^e$, and there are both too many active producers and middlemen: when $\gamma_p \in [D^o, D^e]$, there are too many active producers since $\sigma^e \in [0, 1]$ while $\sigma^o = 0$; when $\gamma_m \in [G^o, G^e]$, there are too many active middlemen since $\tau^e = 1$ while $\tau^o \in [0, 1]$.

Proposition 6. *Given $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} = 1$, the intensive margin is equilibrium is efficient while extensive margin is not. The inefficiency is as shown in Fig.6 in which there are too many producers in the blue shaded areas and too many middlemen in the yellow shaded areas.*

Now we relax the assumption of $\theta_{pc} = \theta_{mc} = 1$ such that the intensive margin is not necessarily efficient. Consider $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} < 1$, then it is easy to check $q_{pm} = q_{pc} \equiv q^e < q^o$, the intensive margin in equilibrium is less than the efficient outcome. For extensive margin, it is proved in the Appendix that efficiency is achieved

for a measure zero set of bargaining power therefore the extensive margin is neither efficient.

Generally, with $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} < 1$, for a given regime of equilibrium, intensive margin (q_{pm}, q_{pc}) are solved by $\alpha_{sb}\theta_{mc}u'(q_{pm}) = c'(q_{pm})$ and $\alpha_{sb}\theta_{pc}u'(q_{pc}) = c'(q_{pc})$, and the extensive margin σ is determined by whether $B^e_{\gamma_p} - \gamma_p$, that is, $\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p$ is positive, negative or zero for a given γ_m. In efficiency, to support the same (τ, σ) as in equlibrium, the extensive margin is determined by whether $B^o_{\gamma_p} - \gamma_p$, that is, $(\alpha_{sb})^2u(q^o) - c(q^o) - \gamma_p$ is positive, negative or zero, and q^o is solved by $\alpha_{sb}u'(q^o) = c'(q^o)$. Graphically, $B^e_{\gamma_p}$ and $B^o_{\gamma_p}$ are the curves dividing $\gamma_p - \gamma_m$ space into regimes. For a given equilibrium outcome of (τ, σ), when $\theta_{pc} = \theta_{mc} = \theta_{sb} < 1$, $q_{pm} = q_{pc} \equiv q^e < q^s$, intensive margin in equilibrium is less than efficiency. For entensive margin, there are two cases to consider. Suppose we want to compare the regimes of a given (τ, σ) in $\gamma_p - \gamma_m$ plane between equilibrium and efficiency. If $\theta_{pc} = \theta_{mc} \geq \alpha_{sb}$, then $\alpha_{sb}\theta_{sb}u(q^o) - c(q^o) > (\alpha_{sb})^2u(q^o) - c(q^o)$. As $\alpha_{sb}\theta_{sb}u(q) - c(q)$ is maximized at $q = q^e$, then $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) > \alpha_{sb}\theta_{sb}u(q^o) - c(q^o)$. Therefore we have $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) > (\alpha_{sb})^2u(q^o) - c(q^o)$, $B^e_{\gamma_p} > B^o_{\gamma_p}$, implying that there are too many active middlemen and producers in equilibrium. If $\theta_{pc} = \theta_{mc} < \alpha_{sb}$, there can be too many or too few active middlemen and producers in equilibrium than in efficiency depending on bargaining powers which affect both intensive margin and extensive margin. For a given (τ, σ), if θ_{sb} is such that $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) > (\alpha_{sb})^2u(q^o) - c(q^o)$, then $B^e_{\gamma_p} > B^o_{\gamma_p}$ implying that there are too many active middlemen and producers; if θ_{sb} is such that $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) < (\alpha_{sb})^2u(q^o) - c(q^o)$, then $B^e_{\gamma_p} < B^o_{\gamma_p}$ implying that there are too few active middlemen and producers. The equilibrium outcome of entensive margin is efficient only if θ_{sb} satisfies $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) = (\alpha_{sb})^2u(q^o) - c(q^o)$, however this is possible only for a measure 0 set of θ_{sb}, therefore we conclude that when $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} < 1$, in equilibrium both intensive and extensive margins are not efficient.

Proposition 7. *Given $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} < 1$, in any outcome of (τ, σ), the intensive margin in equilibrium is always less than the efficient outcome , $q_{pm} = q_{pc} < q^o$. Also the extensive margin is inefficient and there can be too many or too few producers and/or middlemen participating in equilibrium.*

Additionally for the case when $\theta_{pc} = \theta_{mc} < \alpha_{sb}$, we can derive a cutoff curve $\alpha_{sb} = \alpha(\theta_{sb})$ from

$$\begin{cases} \alpha_{sb}\theta_{sb}u(q^e) - c(q^e) & = (\alpha_{sb})^2u(q^o) - c(q^o) \\ \alpha_{sb}\theta_{sb}u'(q^e) & = c'(q^e) \\ (\alpha_{sb})^2u'(q^o) & = c'(q^o) \end{cases}$$

For a given α_{sb}, there is a unique value of θ_{sb} such that $\alpha_{sb}\theta_{sb}u(q^e)-c(q^e) = (\alpha_{sb})^2u(q^o)-c(q^o)$ since the left hand side is increasing in θ_{sb} and right hand side is a constant. Since θ_{sb} is exogenously given while α_{sb} is endogenously determined in response to γ_p and γ_m, there can coexist too many or too few middlemen and producers in $\gamma_p - \gamma_m$ space. For a regime in $\gamma_p - \gamma_m$ space such that $\alpha_{sb} > \alpha(\theta_{sb})$ then are too many middlemen and producers; for a regime of $\gamma_p - \gamma_m$ space such that $\alpha_{sb} < \alpha(\theta_{sb})$ then there are too few. For when there would be too few participation, consider a case when $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} < 1$, then for the regime with $(\tau, \sigma) = (0, 0)$ in Fig.3, we have too few active middlemen or producers. In this regime, $\alpha_{sb} = \bar{\alpha_{sb}} = 1 > \theta_{sb}$. To support $(\tau, \sigma) = (0, 0)$ in equilibrium, γ_p should be more than $\theta_{sb}u(q^e) - c(q^e)$, and in efficiency γ_p should be more than $u(q^o) - c(q^o)$. If we think of q^o to be the value of q^e in equilibrium when $\theta_{sb} = 1$, then because $\frac{\partial\{\theta u[q(\theta)]-c[q(\theta)]\}}{\partial\theta} = \theta u(q) > 0$ and $\theta_{sb} < 1$, then $\theta_{sb}u(q^e) - c(q^e) < u(q^o) - c(q^o)$. We can conclude that when $\gamma_p \in [\theta_{sb}u(q^e) - c(q^e), u(q^o) - c(q^o)]$ and $\gamma_m \leq \theta_{sb}u(q^e) - c(q^e)$, $\tau^e = \tau^o = 0$, while $\sigma^e = 0$ while $\sigma^o = [0, 1]$, implying there are too few active producers in equilibrium. When $\gamma_m \in [\theta_{sb}u(q^e) - c(q^e), u(q^o) - c(q^o)]$ and $\gamma_p < \theta_{sb}u(q^e) - c(q^e)$, $\sigma^e = \sigma^o = 0$, while $\tau^e = [0, 1]$ and $\tau^0 = 0$, implying there are too few active middlmen in equilibrium. When $\gamma_p \in [\theta_{sb}u(q^e) - c(q^e), u(q^o) - c(q^o)]$, $\gamma_m \in [\theta_{sb}u(q^e) - c(q^e), u(q^o) - c(q^o)]$ and $\gamma_m < \gamma_p$, $(\tau^e, \sigma^e) = ([0, 1], 0)$ while $(\tau^o, \sigma^o) = (1, [0, 1])$, implying there are both too few middlemen and producers in equilibrium. Alternative, if $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} = 0$, there would also be too few participation since there is no bargaining power for middlemen and producers who bear sunk costs of having inventory before making a sale in retail market.

The comparison between equilibrium and efficiency on intensive margin and extensive margin sheds light on how bargaining power should be set such that equilibrium is efficient. For the intensive margin, it suggests $\theta_{pc} = \theta_{mc} = 1$ like Lagos and Wright, that is, producers and middlemen who are held up by consumers should have full bargaining power. For extensive margin, it suggests $\theta_{pc} = \theta_{mc} = \theta_{sb} = \frac{\partial m(n_s,n_b)}{\partial_s}\frac{n_s}{n_b} = \alpha_{sb}$ like Hosios, that is, the bargaining power of producers(middlemen) should reflect the elasticity of matching function contributed by their participation.

Obviously it is impossible to have efficiency in extensive and intensive margin by only using bargaining power since it can be set to correct only one of the two inefficiencies at a time. How can we get efficiency for general $(\theta_{mp}, \theta_{mc}, \theta_{pc})$? We propose that proportional subsidies on trading quantities (q_{pm}, q_{pc}) and lump-sum taxes or subsidies on middlemen and producers' participation can be one approach.

Suppose we are choosing proportional taxes (t_p, t_m) and lump-sum taxes (T_p, T_m) such that $(\tau^e, \sigma^e) = (\tau^o, \sigma^o)$ and $(q^e_{pm}, q^e_{pc}) = (q^o_{pm}, q^o_{pc})$. To begin with the analysis, consider proportional subsidies (t_p, t_m) on intensive margin given extensive margin is efficient. Suppose we want to support (τ^o, σ^o). In order to give producers and middle-

men incentive such that they will carry (q^o_{pm}, q^o_{pc}) to RM, which is more than (q^e_{pm}, q^e_{pc}), (t_p, t_m) should satisfy

$$\begin{aligned} t_p &= (1-\theta_{pc})\alpha^o_{sb}u'(q^o) \\ t_m &= (1-\theta_{mc})\alpha^o_{sb}u'(q^o) \end{aligned}$$

where q^o is solved by $\alpha^o_{sb}u'(q^o) = c'(q^o)$, and $t_p > 0$, $t_m > 0$.

Next consider lump-sum taxes(or subsidy) (T_p, T_m) given intensive margin is efficienct with proportional taxes. To support (τ^o, σ^o) as an equilibrium outcome, (T_p, T_m) are given by

$$\begin{aligned} T_p &= (\alpha^o_{sb} - \theta_{pc})\alpha^o_{sb}u(q^o) \\ T_m &= (\alpha^o_{sb} - \theta_{mc})\alpha^o_{sb}u(q^o) \end{aligned}$$

Moreover T_p is a subsidy if $\alpha^o_{sb} > \theta_{pc}$, and a tax if $\alpha^o_{sb} < \theta_{pc}$. Similarly T_m is a subsidy if $\alpha^o_{sb} > \theta_{mc}$, and a tax if $\alpha^o_{sb} < \theta_{mc}$.

6 Effects of Intermediation on Welfare and Redistribution

In this section, I consider effects of intermediation on welfare and redistribution. Specifically, I consider intermediation as a rent extraction activity, in the sense that middlemen have a higher bargaining power than producers when trading with consumers. To this end, I compare welfare and its distribution in two economies: one is an economy with only producers and no middlemen; the other is an economy with middlemen who have a higher bargaining power than producers when trading with consumers in the retail market, i.e $\theta_{mc} > \theta_{pc}$, given the same search cost, i.e. $\gamma_m = \gamma_p$. By the equilibrium results shown in Fig.2 in section 4, middlemen would emerge endogenously with a bargaining power advantage for the reason that they are better than producers at extracting rents from consumers. To simplify notations, denote $\gamma \equiv \gamma_p = \gamma_m$ since search cost is the same for producers and middlemen.

In subsections 7.1 and 7.2, I study the welfare and distribution of a non-intermediated economy and an intermediated economy. I find that, total welfare and each consumer's welfare are improving in quantity per trade per meeting. Increase in the number of sellers can create welfare gains by promoting the number of trades but also welfare losses by incurring higher total costs paid on production and search. I show the ef-

fect of entry decisions on the number and composition of sellers with increasing search cost: when search cost is small, the number of sellers are the same in both economies, and as search cost becomes larger, there are more sellers in the intermediated economy than the non-intermediated. In subsection 7.3, I study the effects of intermediation on welfare and redistribution by comparing the total welfare and distribution of the two economies. I find that, in the view of intermediation as a rent extraction activity, economy can be better with intermediation, in which all types of agents are better off as well. These results can shed new light on this debate and related issues – about middlemen contributing to efficiency versus acting as "vampires" that only buy low and sell high without contributing value added.

6.1 Non-intermediated Economy

We first look for the equilibrium outcome in a non-intermediated economy, i.e. when the economy only have producers and no middlemen. Since middlemen are not available, the wholesale market is removed and retail market is the only decentralized market. Also since producers are the only sellers if the retail market is open, buyer-seller ratio and meeting rates would only change with producers's participation. Let $\hat{\sigma}$ denote producers' participation decision, $\hat{\alpha_{sb}}$ the probability a seller (i.e. producer) meets a buyer (i.e. consumer), $\hat{n_s}$ the number of producers participating in the retail market, and ($\hat{q_{pc}}$,$\hat{y_{pc}}$) terms of trade. Then $\hat{n_s} = \hat{\sigma} N_p$, $\hat{\alpha_{sb}} = \alpha_{sb}(\frac{N_c}{\hat{n_s}}) = (\frac{N_c}{\hat{n_s}})^{\alpha}$. Similar to the decision rule of σ in Section 3, $\hat{\sigma}$ is given by

$$\hat{\sigma} = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \quad if \ \hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}}) - \gamma \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \tag{46}$$

where $\hat{q_{pc}}$ is given by $\hat{\alpha_{sb}}\theta_{pc}u'(\hat{q_{pc}}) = c'(\hat{q_{pc}})$. Note that similar to the analysis in Section 3, $\hat{q_{pc}}$ is decided by the product of $\hat{\alpha_{sb}}$ and θ_{pc}, so $\hat{q_{pc}} = q_{pc}(\hat{\alpha_{sb}}\theta_{pc})$, and $\hat{y_{pc}} = \theta_{pc}u(\hat{q_{pc}})$.

We find that

$$\hat{\sigma} = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \quad if \ \gamma \in \begin{cases} [0, C] \\ [C, H] \\ [H, +\infty) \end{cases} \tag{47}$$

in which C and H are defined in (23) and (21) in Section 4. Then for a given γ, an equilibrium for this economy without intermediation is given by ($\hat{\sigma}$,$\hat{q_{pc}}$,$\hat{y_{pc}}$). The equilibrium exists and is unique.

Now consider the total welfare and distribution in the economy. Let $\hat{W}(\theta_{pc}, \gamma)$

denote the total welfare, $\hat{W}_i(\theta_{pc}, \gamma)$ the welfare for an agent of type $i \in \{C, P\}$, then

$$\hat{W}(\theta_{pc}, \gamma) = (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}}) + \gamma] \tag{48}$$

in which the first term represents welfare gains given by the number of trade $(n_b)^{\alpha}(\hat{n_s})^{1-\alpha}$ and utility created per trade $u(\hat{q_{pc}})$, the second terms is for the welfare cost given by the number of producers $\hat{n_s}$, and production and search costs, $c(\hat{q_{pc}}) + \gamma$, paid by each producer before trading. Welfare for each consumer and producer are given by,

$$\hat{W}_c(\theta_{pc}, \gamma) = (\frac{N_c}{\hat{n_s}})^{\alpha-1}(1 - \theta_{pc})u(\hat{q_{pc}}) \tag{49}$$

$$\hat{W}_p(\theta_{pc}, \gamma) = (\frac{N_c}{\hat{n_s}})^{1-\alpha}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}}) - \gamma \tag{50}$$

in which again $\hat{q_{pc}}$ is endogenously determined by $(\frac{N_c}{\hat{n_s}})^{1-\alpha}\theta_{pc}u'(\hat{q_{pc}}) = c'(\hat{q_{pc}})$.

Lemma 11. *$\hat{W}(\theta_{pc}, \gamma)$, $\hat{W}_c(\theta_{pc}, \gamma)$ are increasing in $\hat{q_{pc}}$.*

Depending on γ, equilibrium outcome of $\hat{\sigma}$ is different so we calculate welfare accordingly. When $\gamma \in [0, C]$, then $\hat{\sigma} = 1$, $\hat{n_s} = N_p$, $\hat{\alpha_{sb}} = (\frac{N_c}{N_p})^{\alpha}$, $\hat{q_{pc}} = q_{pc}[(\frac{N_c}{N_p})^{\alpha}\theta_{pc}]$, then welfare and distribution are given by

$$\hat{W}(\theta_{pc}, \gamma \in [0, C]) = (N_c)^{\alpha}(N_p)^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}}) + \gamma] \tag{51}$$

$$\hat{W}_c(\theta_{pc}, \gamma \in [0, C]) = (\frac{N_c}{N_p})^{\alpha-1}(1 - \theta_{pc})u(\hat{q_{pc}}) \tag{52}$$

$$\hat{W}_p(\theta_{pc}, \gamma \in [0, C]) = (\frac{N_c}{N_p})^{1-\alpha}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}}) - \gamma \tag{53}$$

When $\gamma \in [C, H]$, then $\hat{\sigma} = [0, 1]$, $\hat{n_s} = \hat{\sigma}N_p$, $\hat{\alpha_{sb}} = (\frac{N_c}{\hat{\sigma}N_p})^{\alpha}$, $\hat{q_{pc}} = q_{pc}[(\frac{N_c}{\hat{\sigma}N_p})^{\alpha}\theta_{pc}]$, then welfare and distribution are given by

$$\hat{W}(\theta_{pc}, \gamma \in [C, H]) = (N_c)^{\alpha}(\hat{\sigma}N_p)^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}}) + \gamma] \tag{54}$$

$$\hat{W}_c(\theta_{pc}, \gamma \in [C, H]) = (\frac{N_c}{\hat{\sigma}N_p})^{\alpha-1}(1 - \theta_{pc})u(\hat{q_{pc}}) \tag{55}$$

$$\hat{W}_p(\theta_{pc}, \gamma \in [C, H]) = (\frac{N_c}{\hat{\sigma}N_p})^{1-\alpha}\theta_{pc}u(\hat{q_{pc}}) - -c(\hat{q_{pc}}) - \gamma \tag{56}$$

When $\gamma \in [H, +\infty)$, then $\hat{\sigma} = 0$, $\hat{n_s} = 0$, no producer participates therefore the retail market shuts down and welfare is zero.

6.2 Intermediated Economy

Now by introducing middlemen into the non-intermediated economy, I consider an intermediated economy . As we shut down any difference in search cost and focus on the difference in bargaining powers between producers and middlemen, now intermediation can be regarded as a rent extraction activity. I want to check effects on welfare and redistribution of intermediation.

The advantage in bargaining power endogenously generates intermediation as shown in the Fig.3 in $\gamma_p - \gamma_m$ space with $\theta_{mc} > \theta_{pc}$ in Section 4. As we assume $\gamma_p = \gamma_m = \gamma$, the set of potential equilibria is given by a 45^0 line through the origin in Fig.3. There are four candidate equilibrium outcomes, but depending the bargaining ratio $\frac{\theta_{mc}}{\theta_{pc}}$, there can be two cases as shown in Fig.7.

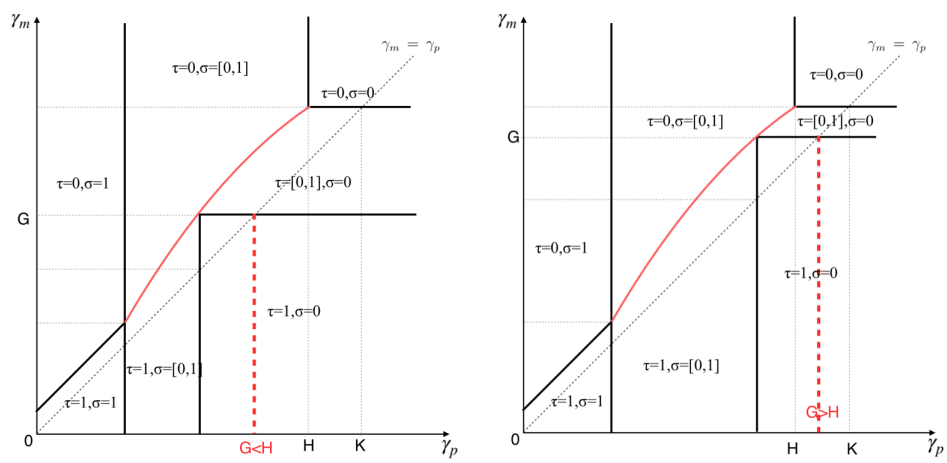

Figure 7:

The left graph shows the equilibrium outcome when $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$, and the right graph shows when $\frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})} < \frac{\theta_{mc}}{\theta_{pc}}$, in which again $\bar{\alpha_{sb}}$ is the highest possible value of α_{sb}. The difference is given by different relationships between G and H, defined in (29) and (21). The left graph is for $H \geq G \equiv G_1$ and in the right graph for $H < G \equiv G_2$. Consider an example that shows why this difference matters. Given $\gamma = H$, the equilibrium in the left panel is $(\tau, \sigma) = ([0,1], 0)$ while in the right is $(\tau, \sigma) = (1, 0)$, clearly τ is higher in the latter for the same γ. This is because in the latter the bargaining power ratio is higher in favor of middlemen and encourages middlemen's participation. This generates different patterns of equilibrium with respect to γ. Now we charaterize equilibrium for the two cases. For $\gamma \in [0, D]$, equilibria are the same for both cases: $(\tau, \sigma) = (1, 1)$ for $\gamma \in [0, C]$, and $(\tau, \sigma) = (1, [0, 1])$ for $\gamma \in [C, D]$. Then they become different for $\gamma > D$: when $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$, $(\tau, \sigma) = (1, 0)$ for $\gamma \in [D, G_1]$, $(\tau, \sigma) = ([0, 1], 0)$ for $\gamma \in [G_1, K]$; when $\frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})} < \frac{\theta_{mc}}{\theta_{pc}}$, $(\tau, \sigma) = (1, 0)$ for $\gamma \in [D, G_2]$, $(\tau, \sigma) = ([0, 1], 0)$ for $\gamma \in [G_2, K]$. Note $K = I$ by $\gamma_m = \gamma_p$, and I is

defined in (20).

Now we consider the welfare and its distribution in this economy. Let $W(\theta_{mc}, \theta_{pc}, \gamma)$ denote the total welfare and $W_i(\theta_{mc}, \theta_{pc}, \gamma)$ denote the welfare for an agent of type $i \in \{C, P, M\}$. Again the number of sellers in the retail market is $n_s = n_p^R + n_m^R = N_p\sigma(1 - \alpha_{pm}\tau) + N_m\alpha_{mp}\tau$. Then total welfare and individual welfare are given by outcomes in the retail market. For total welfare,

$$\begin{aligned} W(\theta_{mc}, \theta_{pc}, \gamma) &= (N_c)^{1-\alpha}(n_s)^{\alpha}\{\frac{n_p^R}{n_s}u(q_{pc}) + \frac{n_m^R}{n_s}u(q_{mc}) \\ &\quad -n_s[\frac{n_p^R}{n_s}c(q_{pc}) + \frac{n_m^R}{n_s}c(q_{mc}) + \gamma] \end{aligned} \tag{57}$$

in which the first term is for welfare gains and the second for welfare losses. For individual welfare,

$$W_c(\theta_{mc}, \theta_{pc}, \gamma) = (\frac{N_c}{n_s})^{\alpha-1}[\frac{n_p^R}{n_s}(1 - \theta_{pc})u(q_{pc}) + \frac{n_m^R}{n_s}(1 - \theta_{mc})u(q_{pm})] \tag{58}$$

$$W_p(\theta_{mc}, \theta_{pc}, \gamma) = (\frac{N_c}{n_s})^{1-\alpha}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma + \alpha_{pm}\theta_{pm}\Sigma_{pm} \tag{59}$$

$$W_m(\theta_{mc}, \theta_{pc}, \gamma) = \alpha_{mp}(1 - \theta_{pm})\Sigma_{pm} \tag{60}$$

in which again α_{pm} and α_{mp} are meeting rates in the wholesale market define in (5) and (6), q_{pc} and q_{pm} in (16) and (17), and Σ_{pm} in (20). Clearly total welfare and its distribution depend on numbers of sellers, proportions of middlemen among sellers and quantity per trade.

Lemma 12. *$W(\theta_{mc}, \theta_{pc}, \gamma)$, $W_c(\theta_{mc}, \theta_{pc}, \gamma)$ are increasing in q_{pc} and q_{pm}.*

No matter $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})}$ or $\frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})} < \frac{\theta_{mc}}{\theta_{pc}}$, the same four candidates of equilibrium exist as γ increases, so we can just derive welfare and its distribution of the four candidates and then match them with different cases of $\frac{\theta_{mc}}{\theta_{pc}}$. First consider $(\sigma, \tau) = (1, 1)$. Then $n_p^R = N_p(1 - \alpha_{pm})$, $n_m^R = N_m\alpha_{mp}$, $n_s = N_p$, $\alpha_{sb} = (\frac{N_c}{N_p})^{\alpha}$, $q_{pc} = q_{pc}(\alpha_{sb}\theta_{pc}) = q_{pc}[(\frac{N_c}{N_p})^{\alpha}\theta_{pc}]$, $q_{pm} = q_{pm}(\alpha_{sb}\theta_{mc}) = q_{pm}[(\frac{N_c}{N_p})^{\alpha}\theta_{mc}]$, $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})]$. Note $N_m\alpha_{mp} = N_p\alpha_{pm}$ is given by the fact that, the number middlemen meet producer, $\tau N_m\alpha_{mp}$, must equal to the number producers meet middlemen $\tau N_p\alpha_{pm}$, so $N_m\alpha_{mp}$ is substituted with $N_p\alpha_{pm}$ in calculation. Now total and

individual welfare are given by

$$
\begin{aligned}
W(\theta_{mc},\theta_{pc},\gamma &\in [0,C]) = (N_c)^{1-\alpha}(N_p)^{\alpha}\{(1-\alpha_{pm})u(q_{pc})+\alpha_{pm}u(q_{mc})\} - N_p[(1-\alpha_{pm})c(q_{pc})+\alpha_{pm}c(\\
W_c(\theta_{mc},\theta_{pc},\gamma &\in [0,C]) = (\frac{N_c}{N_p})^{\alpha-1}[(1-\alpha_{pm})(1-\theta_{pc})u(q_{pc})+\alpha_{pm}(1-\theta_{mc})u(q_{pm})]\\
W_p(\theta_{mc},\theta_{pc},\gamma &\in [0,C]) = (\frac{N_c}{N_p})^{1-\alpha}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma + \alpha_{pm}\theta_{pm}\Sigma_{pm}\\
W_m(\theta_{mc},\theta_{pc},\gamma &\in [0,C]) = \alpha_{pm}(1-\theta_{pm})\Sigma_{pm}
\end{aligned}
$$

Now consider $(\sigma,\tau) = (1,[0,1])$. We have $n_p^R = \sigma N_p(1-\alpha_{pm})$, $n_m^R = N_m\alpha_{mp}$, $n_s = \sigma N_p N_p(1-\alpha_{pm}) + N_m\alpha_{mp}$, $\alpha_{sb} = (\frac{N_c}{n_s})^{\alpha}$, $q_{pc} = q_{pc}(\alpha_{sb}\theta_{pc}) = q_{pc}[(\frac{N_c}{n_s})^{\alpha}\theta_{pc}]$, $q_{pm} = q_{pm}(\alpha_{sb}\theta_{mc}) = q_{pm}[(\frac{N_c}{n_s})^{\alpha}\theta_{mc}]$, $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})]$. Total and individual welfare are derived the same way by (57)-(59). In this case, the composition of sellers is tilted towards middlemen further than the last case, so the probability is higher for a consumer to meet a seller who is a middlemen. Also, q_{pc} and q_{pm} are higher than in the last case since they are increasing in α_{sb} , which increases in a reduction in the number of sellers. Now consider $(\tau,\sigma) = (1,0)$. $n_p^R = 0$, $n_s = n_m^R = N_m\alpha_{mp} = N_p\alpha_{pm}$, $q_{pc} = 0$, $q_{pm} = q_{pm}[(\frac{N_c}{N_p\alpha_{pm}})^{\alpha}\theta_{mc}]$, $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm})$. In this case all producers participate only in the wholesale market and stop entering the retail market so that middlemen are the only sellers. Also q_{pm} is higher than in the last two cases. Lastly, consider $(\tau,\sigma) = [(0,1),0]$. We have $n_p^R = 0$, $n_s = n_m^R = \tau N_m\alpha_{mp} = \tau N_p\alpha_{pm}$, $q_{pc} = 0$, $q_{pm} = q_{pm}[(\frac{N_c}{\tau N_p\alpha_{pm}})^{\alpha}\theta_{mc}]$, $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm})$. In this case n_s is further reduced than in the last case, therefore q_{pm} is even higher.

6.3 Welfare and Redistribution

We now in a position to compare the welfare of these two economies and study the effect of intermediation on welfare redistribution. Again welfare analysis is based on different ranges of γ. Let $\hat{W}(\theta_{pc},\hat{\sigma}) \equiv W(\theta_{pc},\gamma)$ relabel welfare in the non-intermediated economy, where $\hat{\sigma}$ is the participation decision at a given γ, and $W(\theta_{mc},\theta_{pc},\tau,\sigma)$ in the intermediated economy, where $\tau$and σ are the participation decisions for a given γ. Before we start, recall $\frac{\partial q_{pm}}{\partial \alpha_{sb}} \geq 0$, $\frac{\partial q_{pc}}{\partial \alpha_{sb}} \geq 0$, $\frac{\partial q_{pm}}{\partial \theta_{pm}} \geq 0$, $\frac{\partial q_{pc}}{\partial \theta_{pc}} \geq 0$.

Lemma 13. $\frac{\partial[\alpha_{sb}u(q)-c(q)]}{q} \geq 0$, *for* $q \in \{q_{pc}, q_{pm}\}$.

For $\gamma \in [0,C]$, in the non-intermediated economy $\hat{\sigma} = 1$ and in the intermediated economy $(\tau,\sigma) = (1,1)$. So we have $\hat{n_s} = n_s = N_p$ and $\hat{\alpha_{sb}} = \alpha_{sb}$. Also by $\frac{\partial q_{ij}}{\partial(\alpha_{sb}\theta_{ij})} > 0$,

$ij \in \{pc, pm\}$ and $\theta_{mc} > \theta_{pc}$, $\hat{q_{pc}} = q_{pc} < q_{pm}$. This implies the number sellers, therefore buyer-seller ratio and the extensive margin, i.e. number of trades are the same. But the expected welfare in the non-intermediated economy is less than that in the intermediated economy. This is because in the former, the quantity per trade is always $\hat{q_{pc}}$, while in the latter there are some intermediated trade with a larger volume of q_{pm}, so the expected welfare is higher in the latter. Formally, the difference in welfare between the two economies is

$$\begin{aligned} & W(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 1) - \hat{W}(\theta_{pc}, \hat{\sigma} = 1) \\ = \quad & (N_c)^{1-\alpha}(n_s)^{\alpha}\{\alpha_{sb}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}u(q_{pc}) - c(q_{pc})]\} \quad > 0 \end{aligned}$$

It is positive by Lemma 13, with $q_{pm} > q_{pc}$ by $\theta_{mc} > \theta_{pc}$. Therefore total welfare in the intermediated economy is higher than in the non-intermediated. For welfare redistribution, we have

$$W_p(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 1) - \hat{W}_p(\theta_{pc}, \hat{\sigma} = 1) = \alpha_{pm}\theta_{pm}\Sigma_{pm} > 0$$

$$W_m(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 1) = \alpha_{mp}\theta_{mp}\Sigma_{pm} > 0$$

$$\begin{aligned} & W_c(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 1) - \hat{W}_c(\theta_{pc}, \hat{\sigma} = 1) \\ = & (\tfrac{N_c}{N_p})^{\alpha-1}\alpha_{pm}[(1 - \theta_{mc})u(q_{pm}) - (1 - \theta_{pc})u(q_{pc})] \end{aligned}$$

where $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})]$. As shown above, producers and middlemen are better off in the intermediated economy. However for consumers, it is not determined. Intuitively, in the intermediated economy, consumers' welfare gains are created by a larger expected total surplus since upon meeting a middleman they can enjoy q_{pm}, however there are also welfare losses associated with a lower share $1 - \theta_{mc}$ of total surplus. Depending on parameter values, the welfare gains can outweigh the losses.

For $\gamma \in [C, D]$, in the non-intermediated economy $\hat{\sigma} = [0, 1]$ and in the intermediated $(\tau, \sigma) = (1, [0, 1])$. $n_s = N_p\sigma(1 - \alpha_{pm}) + N_m\alpha_{mp}$, and $\hat{n_s} = \hat{\sigma}N_p$. Given the number of sellers in each market and $\theta_{mc} > \theta_{pc}$, we have $\hat{\alpha_{sb}} = \alpha_{sb}$, $\hat{q_{pc}} = q_{pc} < q_{pm}$, $n_s = \hat{n_s}$ and $\sigma < \hat{\sigma}$, proved in the Appendix. Intuitively, as search cost increases, less producers participate in both economies while all middlemen participate because of their higher bargaining power. Moreover, in the intermediated economy, the reduction of producers is larger than in the non-intermediated one for two reasons: one is that producers in the former have an option to sell goods to middlemen when search cost climbs up, the other is that the retail market would be too crowded to be profitable on the sellers' side if $\sigma < \hat{\sigma}$, given middlemen take part of the positions. $\sigma < \hat{\sigma}$ implies that, while $n_s = \hat{n_s}$,

the composition of sellers is tilted towards middlemen compared the case of $\gamma \in [0, C]$, in the sense that among all sellers, a larger proportion of them become middlemen as search cost increases. Then there is a higher chance for consumers to meet a seller who is a middleman and enjoy a larger volume of goods than in a meeting with producers. Similar to the last case, formally welfare change and redistribution are given by

$$\begin{aligned} &W(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = [0,1]) - \hat{W}(\theta_{pc}, \hat{\sigma} = [0,1]) \\ &= (N_c)^{1-\alpha}(n_s)^{\alpha}\{\alpha_{sb}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}u(q_{pc}) - c(q_{pc})]\} > 0 \end{aligned}$$

$$W_p(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = [0,1]) - \hat{W}_p(\theta_{pc}, \hat{\sigma} = [0,1]) = \alpha_{pm}\theta_{pm}\Sigma_{pm} > 0$$

$$W_m(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = [0,1]) = \alpha_{mp}\theta_{mp}\Sigma_{pm} > 0$$

$$\begin{aligned} &W_c(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = [0,1]) - \hat{W}_c(\theta_{pc}, \hat{\sigma} = [0,1]) \\ &= (\tfrac{N_c}{n_s})^{\alpha-1}\{\alpha_{pm}(1-\theta_{mc})u(q_{pm}) - [1-\sigma(1-\alpha_{pm})](1-\theta_{pc})u(q_{pc})\} \end{aligned}$$

where $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})]$. Again, welfare is improved with intermediation, producers and middlemen are better off, consumers can be better off, worse off depending on parameter values.

In the above two cases, both economies has the same number of sellers in the retail market and each producer brings the same quantity of goods. Welfare is higher in the intermediated economy because among all sellers some are middlemen who bring a larger volume of goods, and welfare is increasing in the quantity per trade in equilibrium. This justifies the important role of intensive margin introduced in models of intermediation. Also by $\frac{\partial q_{ij}}{\partial(\alpha_{sb}\theta_{ij})} > 0$, $ij \in \{pc, pm\}$, in these two cases the difference between q_{pm} and $\hat{q_{pc}}$ is only generated by the difference between θ_{mc} and θ_{pc}.

Now for $\gamma > D$, we need to consider the discussion of $\frac{\theta_{mc}}{\theta_{pc}}$ in Section 7.2 because it would generate different pairs of equilibrium to be compared.

We first consider the case when $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})}$ and $H > G \equiv G_1$. For $\gamma \in [D, G_1]$, in the non-intermediated economy $\hat{\sigma} = [0,1]$ and in the intermediated $(\tau, \sigma) = (1, 0)$. Now in the latter economy, producers stop entering and middlemen are the only sellers in the retail market. $n_s = N_m\alpha_{mp}$, $\hat{n_s} = \hat{\sigma}N_p$. Also $n_s > \hat{n_s}$, $\alpha_{sb} \leq \hat{\alpha_{sb}}$, $\hat{q_{pc}} \leq q_{pm}$, proved in Appendix. Intuitively, $\sigma = 0$ in the intermediated economy while $\hat{\sigma} = [0,1]$ in the other indicates that the retail market in the former economy is more crowded for producers to profit from. In fact, the two economies are the same for producers except in the intermediated economy middlemen's participation is promoted by their barganing power advantage and can make the retail market too crowded for producers to make profits. For the quantity per trade q_{pm} and $\hat{q_{pc}}$, as implied by $\frac{\partial q_{ij}}{\partial(\alpha_{sb}\theta_{ij})} > 0$, $ij \in \{pc, pm\}$, here $\theta_{mc} > \theta_{pc}$ while $\alpha_{sb} \leq \hat{\alpha_{sb}}$. It is proved that $\alpha_{sb}\theta_{mc} >$

$\hat{\alpha_{sb}}\theta_{pc}$, indicating that the negative effect of lower meeting rate is outweighed by the positive effect of higher bargaining power in the intermediated economy. Intuitively, $\tau = 1$ implies that middlemen make a positive profit in the intermediated economy, and $\hat{\sigma} = [0, 1]$ indicates producers make a zero free profit in the non-intermediated. Given the same search cost, the reason for a positive profit must be the effect of $\theta_{mc} > \theta_{pc}$ outweighs $\alpha_{sb} \leq \hat{\alpha_{sb}}$. Therefore $\hat{q_{pc}} \leq q_{pm}$. Now we consider total welfare and redistribution. Formally, total welfare in the two economies are given by

$$\begin{aligned} \hat{W}(\theta_{pc}, \hat{\sigma} = [0, 1]) &= (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}}) + \gamma] \\ W(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 0) &= (N_c)^{\alpha}n_s^{1-\alpha}u(q_{mc}) - n_s[c(q_{mc}) + \gamma] \end{aligned}$$

Different from the previous two cases when there is only change in the intensive margin, now both intensive margin, i.e. size of trade, increases from $\hat{q_{pc}}$ to q_{mc}, and extensive margin, i.e. number of producers, increases from $\hat{n_s}$ to n_s. Increase in the intensive margin always creates welfare gains since the holdup problem keeps the quantity per trade inefficiently low in equilibrium. Increase in the extensive margin, however, creates welfare gains as well as losses. As shown above, with a higher number of sellers n_s, welfare is improved by a higher number of meetings but at the cost of a higher sunk payment on search and production. Without specifying parameter values, welfare can be improved, reduced. For welfare redistribution, we found that

$$W_p(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 0) - \hat{W}_p(\theta_{pc}, \hat{\sigma} = [0, 1]) = \alpha_{pm}\theta_{pm}\Sigma_{pm} > 0$$

$$W_m(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 0) = \alpha_{mp}\theta_{mp}\Sigma_{pm} > 0$$

$$\begin{aligned} &W_c(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 0) - \hat{W}_c(\theta_{pc}, \hat{\sigma} = [0, 1]) \\ &= (\tfrac{N_c}{n_s})^{\alpha-1}(1 - \theta_{mc})u(q_{pm}) - (\tfrac{N_c}{\hat{n_s}})^{\alpha-1}(1 - \theta_{pc})u(\hat{q_{pc}}) \end{aligned}$$

So producers and middlemen are better off in the intermediated economy, while consumers can be better off, worse off because while they benefit from a larger volume of goods per trade and a higher meeting rate, they also suffer from a smaller share from total surplus.

Now consider $\gamma \in [G_1, K]$, in the non-intermediated economy $\hat{\sigma} = [0, 1]$ and in the intermediated $(\tau, \sigma) = ([0, 1], 0)$. In fact, compared with the last case, as search cost increases, the total number of sellers reduces further in both economies, and in the intermediated economy some middlemen stop entering now. $n_s = \tau N_m \alpha_{mp}$ and $\hat{n_s} = \hat{\sigma} N_p$. Now $n_s > \hat{n_s}$, $\alpha_{sb} \leq \hat{\alpha_{sb}}$, and $\hat{q_{pc}} = q_{pm}$, proved in Appendix. Intuitively, when search cost becomes larger, middlemen become indifferent between participating or not as they earn a zero profit, same as producers who participate in the non-intermediated

economy. For a given search cost, $n_s > \hat{n_s}$ is generated by middlemen's advantage in bargaining power, and a zero profit is made even though their entry results in a lower buyer-seller ratio than that in the non-intermediated economy. Similar to the last case, total welfare in the two economies are given by

$$\begin{aligned} \hat{W}(\theta_{pc}, \sigma = \hat{[}0,1]) &= (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}}) + \gamma] \\ W(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 0) &= (N_c)^{\alpha}n_s^{1-\alpha}u(q_{mc}) - n_s[c(q_{mc}) + \gamma] \end{aligned}$$

Again, total welfare can be higher or lower with intermediation.

In summary, when $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$, for $\gamma \in [0, D]$, economy is better off with intermediation, and for $\gamma \in [D, K]$, economy can be better or worse off. For any γ, producers and middlemen are better off, while consumers can be better or worse off with intermediation.

Now consider the case when $\frac{\theta_{mc}}{\theta_{pc}} \geq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$ and $H < G \equiv G_2$. For $\gamma \in [D, G_2]$, the results are the same as in the case of $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$ with $\gamma \in [D, G_1]$. For $\gamma \in [G_2, K]$, in the non-intermediated economy $\hat{\sigma} = 0$ and in the intermediated economy $(\tau, \sigma) = ([0, 1], 0)$. This indicates that the retail market is closed in the non-intermediated economy, but still open in the intermediated economy. Clearly, welfare improves with intermediation as it achieves allocations which would not exist in the non-intermediated economy.

In summary, when $\frac{\theta_{mc}}{\theta_{pc}} \geq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$, for $\gamma \in [0, D]$ and $[G, K]$ the economy is better off with intermediation, for $\gamma \in [G, K]$ the economy can be better or worse off. For any γ, producers and middlemen are better off, while consumers can be better or worse off with intermediation.

While change in total and consumers' welfare can have uncertainty with general paramter, I find that with small α in the meeting function $M = (n_b)^{\alpha}(n_s)^{1-\alpha}$, that is sellers' entry dominately decides the nunmber of meetings, then for any γ, economy is better off with intermediation, producers and middlemen are always better off, while consumers are worse when $\gamma \in [0, D]$, and better off when $\gamma > D$. This speaks to the issue of disintermediation and it implies that, when search frictions (i.e. search costs) are small, economy is better off with intermediation while welfare is redistributed such that consumers are worse off, and when search frictions increase, all agents are better off with intermediation.

Proposition 8. *Given $\gamma_m = \gamma_p = \gamma$, and a small α in $M = n_b^{\alpha}n_s^{1-\alpha}$, (1) when γ is small, intermediation improves total welfare, producers and middlemen are better off while consumers are worse off; (2) as γ increases, all agents are better off with intermediation.*

Proposition 9. *Given $\gamma_m = \gamma_p = \gamma$, $\theta_{mc} > \theta_{pc}$, (1) if $\theta_{mc} - \theta_{pc}$ is small, when γ is low, welfare improves with M; as γ increases, welfare can increases, decreases, and for any γ, P and M are weakly better off, C can be better or worse off; (2) if $\theta_{mc} - \theta_{pc}$ is big, when γ is not too big, same as in 1; when γ is large, markets are closed iff M are allowed, agents are better off with intermediation.*

7 Conclusions

This paper has studied the intermediation theory in a search-matching based environment in the spirit of Rubinstein and Wolinsky (1987). The model incorporates divisible goods and an endogenous meeting technology to allow questions related with intensive margin as well as extensive margin to be studied. Compared with models of intermediation with indivisible good, it allows new results to be demonstrated as equilibrium forces can now work in more ways than one. I have proved existence, generic uniqueness of equilibrium and compare it with the efficient outcome. Compared with previous work on middlemen with indivisible goods, I can support a larger set of equilibrium with participation of middlemen. Also in response to changes in parameters, there is co-movement of entensive and intensive margins, and the symmetry of producers and middlemen in their meeting probability in retail market allows advantage (disadvantage) in bargaining power and search cost play an important role in the equilibrium outcome. In the efficiency analysis I find that bargaining power, which has been used in labor and monetary search literature to restore efficiency, is no longer effective here. Since there are inefficiencies on both intensive and extensive margins, if we set bargaining power correctly to have efficiency in one margin we would lose efficiency in the other, thus the paper gives some policy suggestions: for a general set of bargaining power, we should assign proportional subsidy on the intensive margin and lump-sum taxes or subsidy on the extensive margin. Both proportional taxes and lump-sum taxes (subsidies) should be taken into account if there is any difference between producers and middlemen in their bargaining powers. I also explore, without policy intenventions, the effect of intermediation on welfare and redistribution. Taking the view of intermediation as a rent extraction activity, I show that welfare can be improved with intermediation and all agents are better off. This result can speak to the social function of intermediation and issues on disintermediation. All these new findings are given by the economics of divisible good and endogenous meeting technology.

There are many interesting applications to be considered. It would be interesting to allow goods not fully depreciate so that the distribution of inventory over time

can affect agents' decisions and terms of trade. For example, we can allow multiple subperiods of wholesale and retail markets before the settlemen period. It is also desirable to endogenize agents' choice to be producers or middlemen and check changes in the composition of sellers, and the corresponding equilbrium and efficiency results. Furthermore introducing endogenous money or limited credit would be of course within my scope to study issues related with monetary policy and inflation, including cycles and volatility, and in this point of view, middlemen and money can be substitutes but also complements.

Appendix

1. Lemma 1:

Proof. This can be proved by the property of generalized Nash bargaining with perfect credit. It is obvious that M and P would want to trade if each of them gets non-negative surplus from trading. M gets a proportion of θ_{mp} from the total surplus when trading with P, therefore as long as total surplus Σ_{pm} is non-negative, M would be better off trading with P. Similarly, P would want to trade as well if Σ_{pm} is non-negative. Thus the trading decisions for M and P in WM follow the same rule. This can also be proved by considering if there is any profitable deviation for P. Suppose one P chooses not to participate in WM when there are M participating. Then his expected payoff is $V_p = \max\{\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p, 0\} + V_p^A$. If he deviates and trade with M in WM, his expected payoff is $V_p = \alpha_{pm}\theta_{mp}\Sigma_{pm} + \max\{\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p, 0\} + V_p^A$, where α_{sb} are the same if deviating or not since P and M are one-for-one, in the sense that if a marginal P trades with M, the market tightness in retail market is the same as the case if this marginal P goes to RM by himself. Therefore, given M's participation, i.e. $\Sigma_{pm} \geq 0$, if one P deviates to trade with M, all P would trade with M. So as long as M participates in WM, P has an additional chance of getting $\alpha_{pm}\theta_{mp}\Sigma_{pm}$ from participating in WM besides $\max\{\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p, 0\}$ that he gets from participating in RM.

□

2. Lemma 10 and Proposition 1:

Proof. First, we prove $B < C < D < H$ and $G < J$. Recall that

$$
\begin{aligned}
B &= [\alpha_{sb1}\theta_{pc}u(q_{pc1}) - c(q_{pc1})] - [\alpha_{sb1}\theta_{pm}u(q_{pm1}) - c(q_{pm1})] \\
C &= \alpha_{sb1}\theta_{pc}u(q_{pc1}) - c(q_{pc1}) \\
D &= \alpha_{sb2}\theta_{pc}u(q_{pc2}) - c(q_{pc2}) \\
H &= \bar{\alpha_{sb}}\theta_{pc}u(\bar{q_{pc}}) - c(\bar{q_{pc}})
\end{aligned}
$$

where $\alpha_{sb1} = M(1, \frac{N_c}{N_c+N_p})$, $\alpha_{sb2} = M(1, \frac{N_c}{N_c+N_m\alpha_{mp}})$, and $\bar{\alpha_{sb}} = 1$. Obviously $B < C$ since $\alpha_{sb1}\theta_{pm}u(q_{pm1}) - c(q_{pm1}) > 0$.

It is easy to compare C , D and H by checking that in general $\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})$

in monotonically increasing in α_{sb},

$$\begin{aligned}\frac{\partial[\alpha_{sb}\theta_{pc}u(q_{pc})-c(q_{pc})]}{\partial\alpha_{sb}} &= \theta_{pc}[u(q_{pc})+\alpha_{sb}u'(q_{pc})\frac{\partial q_{pc}}{\partial\alpha_{sb}}]-c'(q_{pc})\frac{\partial q_{pc}}{\partial\alpha_{sb}} \\ &= \theta_{pc}u(q_{pc})>0\end{aligned}$$

Since $\alpha_{sb1}<\alpha_{sb2}<\bar{\alpha_{sb}}$ in C, D and H, then $C<D<H$.

Second, we prove that when $\theta_{mc}=\theta_{pc}$ then $A=0$, $F=G$, $J=I$ and $h'(\gamma_p)=1$; when $\theta_{mc}>\theta_{pc}$ then $A>0$, $F<G$, $J<I$ and $h'(\gamma_p)>1$; when $\theta_{mc}<\theta_{pc}$ then $A<0$, $F>G$, $J>I$ and $h'(\gamma_p)<1$. Recall that

$$\begin{aligned}A &= \alpha_{sb1}\theta_{mc}u[q_{pm}(\alpha_{sb1})]-c[q_{pm}(\alpha_{sb1})] \\ &\quad -\{\alpha_{sb1}\theta_{pc}u[q_{pc}(\alpha_{sb1})]-c[q_{pc}(\alpha_{sb1})]\}\end{aligned}$$

For a given α_{sb}, $\frac{\partial\{\alpha_{sb}\theta u[q(\theta)]-c[q(\theta)]\}}{\partial\theta}=\alpha_{sb}u(q)+[\alpha_{sb}\theta u'(q)-c'(q)]\frac{\partial q}{\partial\theta}=\alpha_{sb}u(q)>0$. When $\theta_{mc}>\theta_{pc}$, $\alpha_{sb}\theta_{mc}u[q_{pm}(\theta_{mc})]-c[q_{pm}(\theta_{mc})]>\alpha_{sb}\theta_{pc}u[q_{pc}(\theta_{pc})]-c[q_{pc}(\theta_{pc})]$ therefore given $\alpha_{sb}=\alpha_{sb1}$, $A>0$. Similarly, if $\theta_{mc}<\theta_{pc}$, $A<0$, and if $\theta_{mc}=\theta_{pc}$, $A=0$,

Next consider part $\gamma_m=h(\gamma_p)=\alpha_{sb}\theta_{mc}u(q_{pm})-c(q_{pm})$ in which α_{sb}, q_{pm} can be expressed in terms of γ_p by solving

$$\begin{cases}\gamma_p &= \alpha_{sb}\theta_{pc}u(q_{pc})-c(q_{pc}) \\ c'(q_{pm}) &= \alpha_{sb}\theta_{mc}u'(q_{pm}) \\ c'(q_{pc}) &= \alpha_{sb}\theta_{pc}u'(q_{pc})\end{cases}$$

Taking derivative w.r.t. γ_p on both side of $\gamma_m=h(\gamma_p)$,

$$h'(\gamma_p)=\frac{\partial\alpha_{sb}}{\partial\gamma_p}\{\theta_{mc}u(q_{pm})+[\alpha_{sb}\theta_{mc}u'(q_{pm})-c'(q_{pm})]\frac{\partial q_{pm}}{\partial\alpha_{sb}}\}=\frac{\partial\alpha_{sb}}{\partial\gamma_p}\theta_{mc}u(q_{pm})=\frac{\theta_{mc}u(q_{pm})}{\theta_{pc}u(q_{pc})}$$

Using Proposition 2, when $\theta_{mc}>\theta_{pc}$, then $\theta_{mc}u(q_{pm})>\theta_{pc}u(q_{pc})$, $h'(\gamma_p)>1$, implying $h(\gamma_p)$ is above $f(\gamma_p)$ for $\gamma_p\geq C$, and vice versa. Moreover when $\theta_{mc}>\theta_{pc}$, as $h(\gamma_p)>f(\gamma_p)$ for $\gamma_p\geq C$, then $G=h(\gamma_p=D)>f(\gamma_p=D)=F$, $I=h(\gamma_p=H)>f(\gamma_p=H)=J$. Similarly, when $\theta_{mc}<\theta_{pc}$, as $h(\gamma_p)<f(\gamma_p)$ for $\gamma_p\geq C$, then $G<F$, $I<J$.

Now check the slope and curvature of $\gamma_m=h(\gamma_p)=\gamma_p+\alpha_{sb}(\gamma_p)\theta_{mc}u[q_{pm}(\gamma_p)]-c[q_{pm}(\gamma_p)]-\alpha_{sb}(\gamma_p)\theta_{pc}u[q_{pc}(\gamma_p)]-c[q_{pc}(\gamma_p)]$ for $\gamma_p\in[D,H]$. By $\frac{\partial h(\gamma_p)}{\partial\gamma_p}=\frac{\theta_{mc}u(q_{pm})}{\theta_{pc}u(q_{pc})}$, we have $\frac{\partial h(\gamma_p)}{\partial\gamma_p}>1$ if $\theta_{mc}>\theta_{pc}$, and $0<\frac{\partial h(\gamma_p)}{\partial\gamma_p}<1$ if $\theta_{mc}<\theta_{pc}$. To decide curvature we

check the second order derivative of $h(\gamma_p)$ with respect to γ_p.

$$\begin{aligned}
\frac{\partial^2 h(\gamma_p)}{\partial \gamma_p^2} &= \frac{\partial \frac{\theta_{mc}u(q_{pm})}{\theta_{pc}u(q_{pc})}}{\partial \alpha_{sb}} \frac{\partial \alpha_{sb}}{\partial \gamma_p} \\
&= [\theta_{mc}u'(q_{pm})u(q_{pc}) - \theta_{pc}u'(q_{pc})u(q_{pm})] \frac{\theta_{mc}\theta_{pc}}{[\theta_{pc}u(q_{pc})]^3} \\
&\cong \frac{\partial q_{pm}}{\partial \alpha_{sb}} u'(q_{pm})u(q_{pc}) - \frac{\partial q_{pc}}{\partial \alpha_{sb}} u'(q_{pc})u(q_{pm}) \\
&\cong \frac{u'(q_{pm})}{u(q_{pm})} \frac{\partial q_{pm}}{\partial \alpha_{sb}} - \frac{\theta_{pc}u'(q_{pc})}{u(q_{pc})} \frac{\partial q_{pc}}{\partial \alpha_{sb}}
\end{aligned}$$

Since $\frac{u'(q_{ij})}{u(q_{ij})} \frac{\partial q_{ij}}{\partial \alpha_{sb}}$ is decided by θ_{ij} given other parameters, we just need to check how $\frac{u'(q_{ij})}{u(q_{ij})} \frac{\partial q_{pc}}{\partial \alpha_{sb}}$ response to θ_{ij} to find the sign for $\frac{u'(q_{pm})}{u(q_{pm})} \frac{\partial q_{pm}}{\partial \alpha_{sb}} - \frac{\theta_{pc}u'(q_{pc})}{u(q_{pc})} \frac{\partial q_{pc}}{\partial \alpha_{sb}}$. For simplicity, we drop the subscription for calculation in this step.

$$\begin{aligned}
\frac{\partial [\frac{u'(q)}{u(q)} \frac{\partial q_{pm}}{\partial \alpha_{sb}}]}{\partial \theta} &= \frac{\partial [\frac{u'}{u} \frac{\theta u'}{c''-\alpha\theta u''}]}{\partial \theta} \\
&= \frac{numerator}{[u(c''-\alpha\theta u'')]^2}
\end{aligned}$$

where numerator is

$$\begin{aligned}
&\alpha\theta(u')^2[3u''u - (u')^2] - \theta(u')^2 u(c''' - \alpha\theta u''') \tfrac{\partial q}{\partial \theta} \\
&= \alpha\theta(u')^2[3u''u - (u')^2] - \theta(u')^2 u(c''' - \alpha\theta u''') \tfrac{\alpha u'}{c''-\alpha\theta u''}
\end{aligned}$$

Depending the form of cost and utility functions, $\gamma_m = h(\gamma_p)$ can be concave or convex. But as shown above, if $h(\gamma_p)$ is concave (convex) when $\theta_{mc} > \theta_{pc}$, then it would be convex (concave) when $\theta_{mc} > \theta_{pc}$. We illustrate an example of a concave $h(\gamma_p)$ when $\theta_{mc} > \theta_{pc}$ in the paper.

□

3. Proposition 2:

Proof. Generally $q_{ij} \in \{q_{pc}, q_{mc}\}$ is solved by $\alpha_{sb}\theta_{ij}u'(q_{ij}) = c'(q_{ij})$. Taking derivative w.r.t. α_{sb} on both side, then $\frac{\partial q_{ij}}{\partial \alpha_{sb}} = \theta_{ij} \frac{(u')^2}{c''u'-c'u''} > 0$. Similarly, $\frac{\partial q_{ij}}{\partial \theta_{ij}} = \alpha_{sb} \frac{(u')^2}{c''u'-c'u''} > 0$. Then let η_{ij} denote $\alpha_{sb}\theta_{ij}$, then $\frac{\partial q_{ij}}{\partial \eta_{ij}} = \frac{u'(q_{ij})}{c''(q_{ij})-\eta_{ij}u''(q_{ij})} \geq 0$.

□

4. Proposition 3:

Proof. In equilibrium with $(\tau, \sigma) = (1, [0, 1])$, we have

$$\begin{cases} \gamma_p & = \alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) \\ \gamma_m & \leq \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) \end{cases}$$

where $\alpha_{sb} = M(1, \frac{N_c}{N_p\sigma(1-\alpha_{pm})+N_m\alpha_{mp}})$. Also q_{pm}, q_{pc} and σcan be solved in terms of γ_p by Eq. 32, and

$$\gamma_p = \alpha_{sb}(\sigma)\theta_{pc}u[q_{pc}(\sigma)] - c[q_{pc}(\sigma)] \tag{65}$$

Taking derivative with respect to γ_p on both of Eq.42,

$$\begin{aligned} 1 &= \frac{\partial \alpha_{sb}}{\partial \sigma}\frac{\partial \sigma}{\partial \gamma_p}\{\theta_{pc}u(q_{pc}) + [\alpha_{sb}\theta_{pc}u'(q_{pc}) - c'(q_{pc})]\frac{\partial q_{pc}}{\partial \alpha_{sb}}\} \\ \frac{\partial \sigma}{\partial \gamma_p} &= [\frac{\partial \alpha_{sb}}{\partial \sigma}\theta_{pc}u(q_{pc})]^{-1} < 0 \end{aligned}$$

Moreover $\frac{\partial \alpha_{sb}}{\partial \gamma_p} = \frac{\partial \alpha_{sb}}{\partial \sigma}\frac{\partial \sigma}{\partial \gamma_p} = [\theta_{pc}u(q_{pc})]^{-1} > 0$, $\frac{\partial q_{pc}}{\partial \gamma_p} = \theta_{pc}\frac{(u')^2}{c''u'-c'u''}[\theta_{pc}u(q_{pc})]^{-1} > 0$, $\frac{\partial q_{pm}}{\partial \gamma_p} = \theta_{mc}\frac{(u')^2}{c''u'-c'u''}[\theta_{mc}u(q_{pm})]^{-1} > 0$. Similarly the same results can be proved in equilibrium with $(\tau, \sigma) = (0, [0, 1])$.

□

5. Proposition 5:

Proof. For a social planner, the problem is given by

$$\begin{aligned} \max_{\tau^o, \sigma^o, q^o_{pm}, q^o_{pc}} & N_p\tau^o\alpha^o_{pm}[-c(q^o_{pm})] + N_m\tau^o\alpha_{mp}[\alpha^o_{sb}u(q^o_{pm}) - \gamma_m] \\ & + N_p\sigma^o(1 - \tau^o\alpha_{pm})[\alpha^o_{sb}u(q^o_{pc}) - c(q^o_{pc}) - \gamma_p] \end{aligned} \tag{66}$$

where $\alpha^o_{sb} = M(1, \frac{N_c}{N_p\sigma^o(1-\alpha_{pm}\tau^o)+N_m\alpha_{mp}\tau^o})$. By using $N_p\alpha_{pm}$ to substitute for $N_m\alpha_{mp}$ in the optimization problem and dividing the function by N_p, then it is the same as solving

$$\max_{\tau^o, \sigma^o, q^o_{pm}, q^o_{pc}} Z \equiv \tau^o\alpha_{pm}[\alpha^o_{sb}u(q^o_{pm}) - c(q^o_{pm}) - \gamma_m] + \sigma^o(1 - \tau^o\alpha_{pm})[\alpha^o_{sb}u(q^o_{pc}) - c(q^o_{pc}) - \gamma_p] \tag{67}$$

given α^o_{sb}, $q^o_{pm} = q^o_{pc} = q^o$ is solved by

$$\alpha^o_{sb}u'(q^o) = c'(q^o) \tag{68}$$

$$
\begin{aligned}
\frac{\partial Z}{\partial \tau^o} &= \alpha_{pm}[\alpha^o_{sb}u(q^o) - c(q^o) - \gamma_m] + \tau^o\alpha_{pm}\{\frac{\partial \alpha^o_{sb}}{\partial \tau}u(q^o) + [\alpha^o_{sb}u'(q^o) - c'(q^o)]\frac{\partial q^o}{\partial \alpha^o_{sb}}\frac{\partial \alpha^o_{sb}}{\partial \tau} \\
&\quad -\alpha_{pm}\sigma^o[\alpha^o_{sb}u(q^o) - c(q^o) - \gamma_p] + \sigma^o(1-\tau^o\alpha_{pm})\{\frac{\partial \alpha^o_{sb}}{\partial \tau}u(q^o) + [\alpha^o_{sb}u'(q^o) - c'(q^o)]\frac{\partial q^o}{\partial \alpha^o_{sb}}\frac{\partial \alpha^o_{sb}}{\partial \tau} \\
&= \alpha_{pm}\{\alpha^o_{sb}u(q^o) - c(q^o) - \gamma_m - \sigma^o[\alpha^o_{sb}u(q^o) - c(q^o) - \gamma_p]\} + \frac{\partial \alpha^o_{sb}}{\partial \tau^o}u(q^o)[\alpha_{pm}\tau + \sigma^o(1-\tau\alpha_{pm})] \\
&= 0 \qquad (69)
\end{aligned}
$$

$$
\frac{\partial Z}{\partial \sigma^o} = (1-\tau^o\alpha_{pm})[\alpha^o_{sb}u(q^o) - c(q^o) - \gamma_p] + \frac{\partial \alpha^o_{sb}}{\partial \sigma^o}u(q^o)[\alpha_{pm}\tau + \sigma^o(1-\tau\alpha_{pm})] = 0 \quad (70)
$$

where

$$
\begin{aligned}
\frac{\partial \alpha^o_{sb}}{\partial \tau^o} &= -\frac{N_cN_p\alpha_{pm}(1-\sigma^o)}{[N_c + N_p\sigma^o(1-\alpha_{pm}\tau^o) + N_m\alpha_{mp}\tau^o]^2} \qquad (71) \\
\partial \alpha^o_{sb} &= -\frac{N_cN_p(1-\alpha_{pm}\tau^o)}{[N_c + N_p\sigma^o(1-\alpha_{pm}\tau^o) + N_m\alpha_{mp}\tau^o]^2} \qquad (72)
\end{aligned}
$$

□

6. Extension: Give Producers another chance to trade with consumers after trading with a middleman

Now suppose we relax the assumption that a producer is not eligible to produce again for retail market if he has traded with a middleman and allow him to produce again for consumers. The equilibrium set is as shown in Fig.6 for the cases when $\theta_{pc} > \theta_{mc}$.

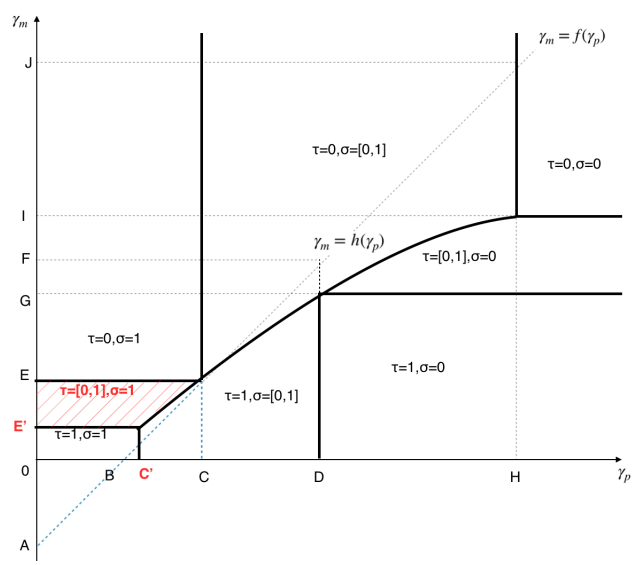

Figure 8:

Here are some interesting observations. The equilibrium set is quite similar to that in the baseline model qualitatively. However there are several differences to notice:

first, there emerges a new regime with $(\tau, \sigma) = ([0,1], 1)$, as shown in the red shaded area, while in baseline model this is a regime with a zero set of parameters. Second, compared with Fig.3 in the baseline model, near origin $\gamma_p = \gamma_m = 0$ now the equilibrium is $(\tau, \sigma) = (1, 1)$ while in the baseline model it is $(0, 1)$. Third, the equilibrium regime with $(\tau, \sigma) = (1, [0, 1])$ is extended such that when $\gamma_p \in [C', C]$, σ is now a mixed strategy instead of 1 implying some but not all producers participate. Last, since producers can produce again, to support an equilibrium in which producers choose to produce again after trading with middlemen, the decision rule for $\tau$is now only determined by the range of γ_m without any need to be compared with γ_p. In the baseline model, this is not the case because once produce choose to trade with middlemen he can avoid the search cost γ_p in retail market. However if a producer choose to produce for retail market when he is given the chance, γ_p will be paid no matter he trades or not with a middlemen therefore γ_p is not considered when evaluating the total surplus of a producer-middleman meeting. All these differences are related to economics of giving producers the chance to producer again.

7. Equilibrium in an economy with only producers and no middlemen in Section 7.1

To start with the equilibrium in this economy, consider a pure strategy equilibrium that $\hat{\sigma} = 1$, which implies $\hat{n_s} = N_p$. This equilibrium is supported by $\gamma \leq \hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}})$, in which $\hat{\alpha_{sb}} = (\frac{N_c}{N_p})^{\alpha}$, and $\hat{q_{pc}} = q_{pc}(\hat{\alpha_{sb}})$, $y_{cp} = \theta_{pc}u[q_{pc}(\hat{\alpha_{sb}})]$. It is easy to show that when $\hat{\alpha_{sb}} = (\frac{N_c}{N_p})^{\alpha}$, $\hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}})$ equals to the value of C in Section 4. Now consider the other pure strategy equilibrium with $\hat{\sigma} = 0$. This equilibrium is supported if $\gamma \geq \hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}})$, in which $\hat{\alpha_{sb}} = \bar{\alpha_{sb}}$ and $\hat{q_{pc}} = q_{pc}(\hat{\alpha_{sb}})$, $y_{cp} = \theta_{pc}u[q_{pc}(\hat{\alpha_{sb}})]$. When $\hat{\alpha_{sb}} = \bar{\alpha_{sb}}$, $\hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}})$ equals to the value of H in Section 4. Lastly, consider the mixed strategy equilbrium with $\hat{\sigma} = [0, 1]$. This equilibrium is supported if $\gamma = \hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}})$, in which $\hat{\alpha_{sb}} = (\frac{N_c}{\hat{\sigma}N_p})^{\alpha}$ and $\hat{q_{pc}} = q_{pc}(\hat{\alpha_{sb}})$, $y_{cp} = \theta_{pc}u[q_{pc}(\hat{\alpha_{sb}})]$. Therefore

$$\hat{\sigma} = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \quad if \ \gamma \in \begin{cases} [0, C] \\ [C, H] \\ [H, +\infty) \end{cases} \tag{73}$$

in which C and H are defined in (23) and (21) in Section 4.

8. Lemma 11:

Proof. Total welfare in a non-intermediated economy is given by $\hat{W}(\theta_{pc}, \gamma)$, then

$$\begin{aligned} \frac{\partial \hat{W}(\theta_{pc}, \gamma)}{\partial q_{pc}} &= \frac{\partial\{(N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}}) + \gamma]\}}{\partial q_{pc}} \\ &= (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u'(\hat{q_{pc}}) - \hat{n_s}c'(\hat{q_{pc}}) \end{aligned}$$

we know that $\hat{\alpha_{sb}}\theta_{pc}u'(\hat{q_{pc}}) = c'(\hat{q_{pc}})$ and $\hat{\alpha_{sb}} = (\frac{N_c}{\hat{n_s}})^{1-\alpha}$, so

$$\begin{aligned}\frac{\partial \hat{W}(\theta_{pc},\gamma)}{\partial q_{pc}} &= (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u'(\hat{q_{pc}}) - \hat{n_s}c'(\hat{q_{pc}}) \\ &= (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}(1-\theta_{pc})u'(\hat{q_{pc}}) \geq 0\end{aligned}$$

Also

$$\frac{\partial \hat{W}_c(\theta_{pc},\gamma)}{\partial q_{pc}} = (\frac{N_c}{\hat{n_s}})^{\alpha-1}(1-\theta_{pc})u'(\hat{q_{pc}}) \geq 0$$

□

9. Lemma 12

Proof. Total welfare in an intermediation economy is given by $W(\theta_{mc},\theta_{pc},\gamma)$, similar as the proof for Lemma 14

$$\frac{\partial W(\theta_{mc},\theta_{pc},\gamma)}{\partial q_{pc}} = (N_c)^{1-\alpha}(n_s)^{\alpha}\frac{n_p^R}{n_s}(1-\theta_{pc})u'(q_{pc}) \geq 0$$

$$\frac{\partial W(\theta_{mc},\theta_{pc},\gamma)}{\partial q_{pm}} = (N_c)^{1-\alpha}(n_s)^{\alpha}\frac{n_m^R}{n_s}(1-\theta_{mc})u'(q_{pm}) \geq 0$$

$$\frac{\partial W_c(\theta_{mc},\theta_{pc},\gamma}{\partial q_{pc}}) = (\frac{N_c}{n_s})^{\alpha-1}\frac{n_p^R}{n_s}(1-\theta_{pc})u'(q_{pc}) \geq 0$$

$$\frac{\partial W_c(\theta_{mc},\theta_{pc},\gamma)}{\partial q_{pm}} = (\frac{N_c}{n_s})^{\alpha-1}\frac{n_m^R}{n_s}(1-\theta_{mc})u'(q_{pm}) \geq 0$$

□

10. Lemma 13

Proof. For net welfare created per trade, $\alpha_{sb}u(q_{ij}) - c(q_{ij}) - \gamma$, for $ij \in \{pc, pm\}$

$$\begin{aligned}\frac{\partial[\alpha_{sb}u(q_{ij}) - c(q_{ij}) - \gamma]}{\partial q_{ij}} &= \frac{\partial[\alpha_{sb}u(q_{ij}) - c(q_{ij})]}{\partial q_{ij}} \\ &= \alpha_{sb}(1-\theta_{ij})u'(q_{ij}) \geq 0\end{aligned}$$

□

11. Proofs for subsection 7.3

Proof. Consider a comparison of $\hat{\sigma} = 1$ in the non-intermediated economy and $(\tau, \sigma) = (1, 1)$ in the intermediated. For extensive margin, since the number of sellers are the same in both economy $\hat{n_s} = n_s = N_p$, then $\hat{\alpha_{sb}} = \alpha_{sb}$. For the intensive margin, by $\hat{q_{pc}}$, q_{pc}, q_{pm} are given by

$$c'(\hat{q_{pc}}) = \hat{\alpha_{sb}}\theta_{pc}u'(\hat{q_{pc}}) \tag{74}$$

$$c'(q_{pc}) = \alpha_{sb}\theta_{pc}u'(q_{pc}) \tag{75}$$

$$c'(q_{pm}) = \alpha_{sb}\theta_{mc}u'(q_{pm}) \tag{76}$$

Given $\hat{\alpha_{sb}} = \alpha_{sb}$, and $\theta_{pc} < \theta_{mc}$, by Proposition 2, we have $\hat{q_{pc}} = q_{pc} < q_{pm}$.

Now consider a comparison between $\hat{\sigma} = [0, 1]$ in the non-intermediated economy and $(\tau, \sigma) = (1, [0, 1])$ in the intermediated. For extensive margin, $\hat{n_s} = \hat{\sigma}N_p$, and $n_s = N_p\sigma(1 - \alpha_{pm}) + N_m\alpha_{mp}$ in which the first term is the number of producers and the second the number of middlemen. For the intensive margin, in order to compare $\hat{q_{pc}}, q_{pc}$ and q_{pm}, we need to compare $\hat{\alpha_{sb}}$ and α_{sb}. Recall the equilibrium conditions on γ to support $\hat{\sigma} = [0, 1]$, and $(\tau, \sigma) = (1, [0, 1])$,

$$\hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}}) = \gamma \tag{77}$$

$$\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) = \gamma \tag{78}$$

$$\alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) \geq \gamma \tag{79}$$

Using (74) though (76), by (77) and (78), we know $\hat{\alpha_{sb}} = \alpha_{sb}$, and $\hat{q_{pc}} = q_{pc} < q_{pm}$. Also by $\hat{\alpha_{sb}} = \alpha_{sb}$, we have $\hat{n_s} = n_s$, $\hat{\sigma} > \sigma$.

Now consider a comparison between $\hat{\sigma} = [0, 1]$ in the non-intermediated economy and $(\tau, \sigma) = (1, 0)$ in the intermediated. For the extensive margin, $\hat{n_s} = \hat{\sigma}N_p$, and $n_s = N_m\alpha_{mp}$, now the intermediated economy only have middlemen as sellers in the retail market. For the intensive margin, recall the equilibrium conditions on γ to support $\hat{\sigma} = [0, 1]$, and $(\tau, \sigma) = (1, 0)$,

$$\hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}}) = \gamma \tag{80}$$

$$\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) \leq \gamma \tag{81}$$

$$\alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) \geq \gamma \tag{82}$$

Again using (74) though (76), by (80) and (81) we know $\hat{\alpha_{sb}} \geq \alpha_{sb}$ and by (80) and (82) we have $\hat{\alpha_{sb}}\theta_{pc} \leq \alpha_{sb}\theta_{mc}$. Since $\frac{\partial q_{ij}}{\partial(\alpha_{sb}\theta_{ij})}$ by Proposition 2, we have $q_{pc} \leq \hat{q_{pc}} \leq q_{pm}$.

Now consider a comparison between $\hat{\sigma} = [0, 1]$ in the non-intermediated economy and $(\tau, \sigma) = ([0, 1], 0)$ in the intermediated. For the extensive margin, $\hat{n_s} = \hat{\sigma}N_p$, and

$n_s = \sigma N_m \alpha_{mp}$. For the intensive margin, by

$$\hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}}) = \gamma \tag{83}$$

$$\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) \leq \gamma \tag{84}$$

$$\alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) = \gamma \tag{85}$$

using (74) though (76), $\hat{\alpha_{sb}} \geq \alpha_{sb}$ from (83) and (84), $\hat{\alpha_{sb}}\theta_{pc} = \alpha_{sb}\theta_{mc}$ from (83) and (85), So $q_{pc} \leq \hat{q_{pc}} = q_{pm}$. Also by $\hat{\alpha_{sb}} \geq \alpha_{sb}$, we know $\hat{n_s} \leq n_s$.

□

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